The Standard Economic Model

PSYC201: Cognitive Psychology

Mark Hurlstone
Lancaster University

Week 9

Decision Making Under Risk and Uncertainty

- Everyday we are faced with decisions (some small, some big):
  - should you take your umbrella to work?
  - should you take out travel insurance?
  - what pension fund should you invest in?
  - what football team should you support?
  - should you believe in God?
- These decisions involves choices between several options concerning “future states of the world” that are “uncertain” or “unknown”
- They involve decision making under risk or uncertainty

The Standard Economic Model: Expected Utility Theory

Daniel Bernoulli, 1700-1782

Blaise Pascal, 1623-1662

Frank Ramsey, 1903-1930

Leonard Savage, 1917-1971

John von Neumann (right; 1903-1957) and Oskar Morgensttern (left; 1902-1977)

1. People choose different possible options (e.g., whether or not to take an umbrella to work) by selecting the option that maximises expected utility
2. More of the same thing (e.g., wealth) creates additional utility with a decreasing rate

Expected Utility Theory

- An individual maximises the expected utility of the outcomes associated with different possible acts, subject to a probability distribution p of the different states S of the world:

$$\underbrace{\boxed{\max}}_{(1)} \underbrace{\boxed{\sum p(S_j)}}_{(2)} \cdot \underbrace{\boxed{U(O_{ij})}}_{(3)}$$

- where:
  - j indexes the different states of the world
  - U is the utility of the outcome O of each act i for each state of the world

Acts, States, and Outcomes

- The set of acts $\{A_i\}$ are the options that decision makers must choose between
  - e.g., take an umbrella to work (A_1) *vs.* leave umbrella at home (A_2)
- The set of states $\{S_j\}$ correspond to various possible ways the world might turn out
  - e.g., rain (S_1) *vs.* no rain (S_2)
- The set of outcomes $\{O_{ij}\}$ are the different possible consequences of each act, given each possible state

Acts, States, and Outcomes

Act	State	Outcome
Take umbrella	Rain	$O_{1,1}$
	No rain	$O_{1,2}$
Leave umbrella	Rain	$O_{2,1}$
	No rain	$O_{2,2}$

Utility

- **Utility** is an index or measure of preference
- The utility of an outcome corresponds to how much the decision maker values that outcome
- A **utility function** associates each possible outcome with a number and expresses a person's preference ordering over the outcomes (ties are allowed)
- Larger values are assigned to more preferred outcomes
- This view of utility is known as **ordinal utility** because it enables you to order preferences

Utility

Table: Payoffs

	Rain	No Rain
Take umbrella	Dry, not happy	Dry, not happy
Leave umbrella	Wet, miserable	Dry, happy

Table: Utility payoffs

	Rain	No Rain
Take umbrella	?	?
Leave umbrella	?	?

Utility

Table: Payoffs

	Rain	No Rain
Take umbrella	Dry, not happy	Dry, not happy
Leave umbrella	Wet, miserable	Dry, happy

Table: Utility payoffs

	Rain	No Rain
Take umbrella	3	3
Leave umbrella	0	5

Utility

Table: Utility payoffs

	Rain	No Rain
Take umbrella	3	3
Leave umbrella	0	5

- The utility function is: $u(O_{1,1})$ = 3, $u(O_{1,2})$ = 3, $u(O_{2,1})$ = 0, $u(O_{2,2})$ = 5, yielding the sequence $\langle$0,3,3,5$\rangle$

Probabilities

- It is assumed individuals derive probabilities for the different possible states of the world (e.g., rain *vs.* no rain)
- Sometimes probabilities are derived *objectively*
  - e.g., consulting a weather forecast to derive the probability of rain *vs.* no rain
- Other times probabilities are derived *subjectively*
  - e.g., observing the clouds to derive the probability of rain *vs.* no rain
- The theory assumes probabilities are bounded by 0 (= definitely will not happen) and 1 (= definitely will happen)
- The value ½ is reserved for a state that is equally likely to happen as not

Maximising Expected Utility

- The key assumption of EUT is that decision makers obey the principle of **maximising expected utility**
- This involves computing the **expected utility** of each act using the EUT formula given earlier
- For each act, the probability of each state, $P(S_j)$, is multiplied by the utility of each outcome $U(O_{ij})$, and the sum of all products gives the expected utility of the act
- The act with the highest expected utility is then selected

Maximising Expected Utility

Act	State	Probability	Utility	Outcome
Take umbrella	Rain	0.6	3	$O_{1,1}$
	No rain	0.4	3	$O_{1,2}$
Leave umbrella	Rain	0.6	0	$O_{2,1}$
	No rain	0.4	5	$O_{2,2}$

Maximising Expected Utility

Act	State	Probability	Utility	Outcome
Take umbrella	Rain	0.6	3	$O_{1,1}$
	No rain	0.4	3	$O_{1,2}$
Leave umbrella	Rain	0.6	0	$O_{2,1}$
	No rain	0.4	5	$O_{2,2}$

$$\begin{aligned} EU(\text{Take umbrella}) &= P(S_1) \cdot U(O_{1,1}) + P(S_2) \cdot U(O_{1,2}) \\ &= 0.6 \cdot 3 + 0.4 \cdot 3 \\ &= 3 \end{aligned}$$

Maximising Expected Utility

Act	State	Probability	Utility	Outcome
Take umbrella	Rain	0.6	3	$O_{1,1}$
	No rain	0.4	3	$O_{1,2}$
Leave umbrella	Rain	0.6	0	$O_{2,1}$
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$$\begin{aligned} EU(\text{Take umbrella}) &= P(S_1)\cdot U(O_{1,1}) + P(S_2)\cdot U(O_{1,2}) \\ &= 0.6\cdot 3 + 0.4\cdot 3 \\ &= 3 \end{aligned}$$

$$\begin{aligned} EU(\text{Leave umbrella}) &= P(S_1)\cdot U(O_{2,1}) + P(S_2)\cdot U(O_{2,2}) \\ &= 0.6\cdot 0 + 0.4\cdot 5 \\ &= 2 \end{aligned}$$

Maximising Expected Utility

Act	State	Probability	Utility	Outcome
Take umbrella	Rain	0.6	3	$O_{1,1}$
	No rain	0.4	3	$O_{1,2}$
Leave umbrella	Rain	0.6	0	$O_{2,1}$
	No rain	0.4	5	$O_{2,2}$

$$\begin{aligned} EU(\text{Take umbrella}) &= P(S_1) \cdot U(O_{1,1}) + P(S_2) \cdot U(O_{1,2}) \\ &= 0.6 \cdot 3 + 0.4 \cdot 3 \\ &= \mathbf{3} \end{aligned}$$

$$\begin{aligned} EU(\text{Leave umbrella}) &= P(S_1) \cdot U(O_{2,1}) + P(S_2) \cdot U(O_{2,2}) \\ &= 0.6 \cdot 0 + 0.4 \cdot 5 \\ &= 2 \end{aligned}$$

Status of The Theory

- EUT is a **normative theory**—it prescribes how people should choose if they are rational decision makers
- It is also widely applied as a **descriptive theory**—it is assumed that people are rational decision makers and that they obey the principles of EUT
- EUT is an **'as-if' theory**—it is assumed people make decisions *as-if* they have certain probability and utility functions, and maximise expected utility
- It does not claim that people actually do this (although some economists would disagree)!

Prospects

- EUT is a theory of decision making under risk
- Decision making under risk is a process of choosing between different **prospects** or gambles
- A prospect consists of a number of possible outcomes along with their associated probabilities
- An example of a decision under risk would involve choosing between the following two prospects
  - **Prospect A**: 50% chance to win 100; 50% chance to win nothing
  - **Prospect B**: Certainty of winning 45

Prospects

- A prospect can be described formally as
  - $\boldsymbol{q} = (x_1, p_1; \ldots x_n, p_n)$
  - where x_i represents the outcomes and p_i represents the associated probabilities
- Prospect A (previous slide) could be represented as $\boldsymbol{q} = (100, 0.5; 0, 0.5)$ or more simply as $(100, 0.5)$
- Prospect B (previous slide) could be represented as $\boldsymbol{r} = (45)$

Rationality Axioms

- The axioms of EUT were developed by von Neumann and Morgenstern (1947)
- Axioms are basic propositions that cannot be proven and must be taken for granted
- They are fundamental to the model
- If people violate the axioms, then this is problematic for the theory

Expected Utility Theory: Axioms

- **Completeness**
  - This requires that for all $\boldsymbol{q}$, $\boldsymbol{r}$:
  - Either $\boldsymbol{q} \succeq \boldsymbol{r}$ or $\boldsymbol{r} \succeq \boldsymbol{q}$ or $\boldsymbol{r} \backsim \boldsymbol{q}$
- **Transitivity**
  - If we take any three prospects, $\boldsymbol{q}$, $\boldsymbol{r}$, $\boldsymbol{s}$
  - if $\boldsymbol{q} \succeq \boldsymbol{r}$ and $\boldsymbol{r} \succeq \boldsymbol{s}$, then $\boldsymbol{q} \succeq \boldsymbol{s}$
  - People sometimes violate this assumption

Dominance

- If prospect ***q*** is better than prospect ***r*** in at least one respect, and at least as good as prospect ***r*** in all other respects, then prospect ***q*** should always be preferred
- People sometimes violate this axiom
- Suppose there are two slot machines: the probability of a fixed payout for the left machine is $P(L) = .7$, whereas the probability for the right machine is $P(R) = .3$
- People engage in *probability matching*—inserting coins in the left machine 70% of the time and into the right machine 30% of the time
- The optimal strategy is to put all your money in the left machine

Invariance

- States that different ways of presenting (framing) the same choice problem should not yield different preferences
- Preferences should be:
  - description invariant: unaffected by the description of the choice options
  - procedure invariant: unaffected by the procedure used to elicit a person's preferences
- **Framing effects** should not happen, but as we will see in the next lecture, they frequently do

Cancellation (Sure-Thing Principle)

- This axiom is fundamental to EUT
- States of the world that give the same outcome regardless of one's choice can be be eliminated (cancelled) from the choice problem
- The **sure-thing principle** (Savage, 1954):

> *If someone would prefer option A to option B if event X occurs, and would also prefer option A to option B if event X does not occur, then they should prefer A to B when they are ignorant of whether or not X occurs*

- Why should your choice between options be affected by events that have no impact on the outcome of interest?

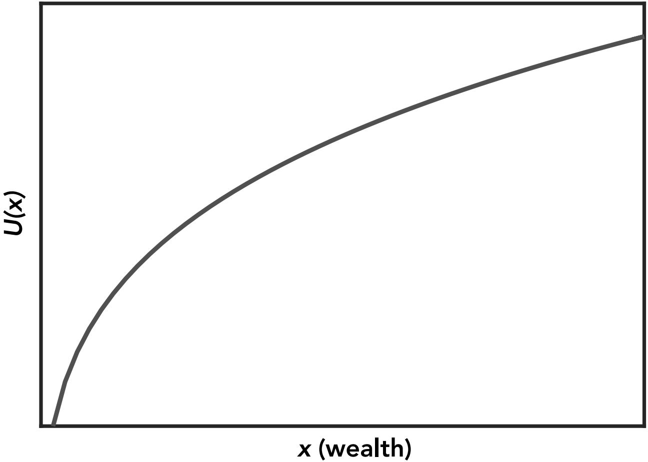
Cancellation (Sure-Thing Principle)

Imagine that a businessman is considering whether or not to buy a property. The businessman thinks that the attractiveness of this purchase will depend in part on the result of the upcoming presidential election. To clarify things he asks himself whether he would buy if he knew that the Republican candidate would win, and decides he would. He then asks himself whether he would buy if he knew that the Democratic candidate would win, and again decides that he would. Given that he would buy the property in either event, this is the appropriate action even though he does not know what the result of the election will be.

Risk Preferences and Utility Functions

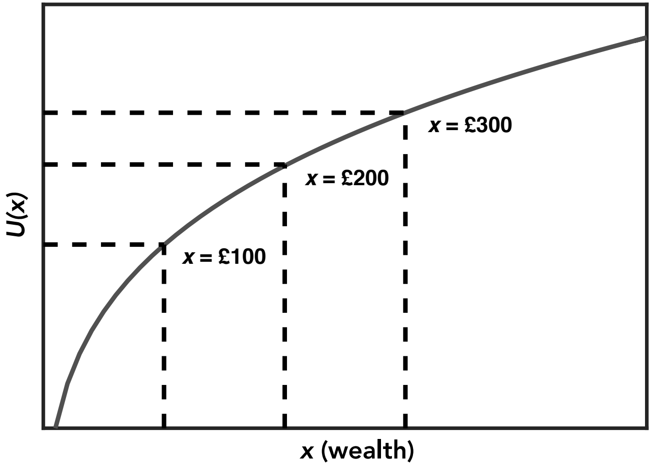
- The utilities that people assign to outcomes capture their **risk preferences**
- EUT (typically) assumes that people are *risk averse*
- A person is risk averse if they would reject a gamble in favour of a sure amount equal to its expected value
- For example, most people would prefer £500 than a 50–50 chance of £1000
- To capture people's risk preferences, EUT incorporates a **utility function** that converts objective values of some quantity (e.g., money) into subjective utility values

Risk Aversion



- The EUT utility function conforms to a power function of the form $u = x^b$, where $b < 1$

Risk Aversion



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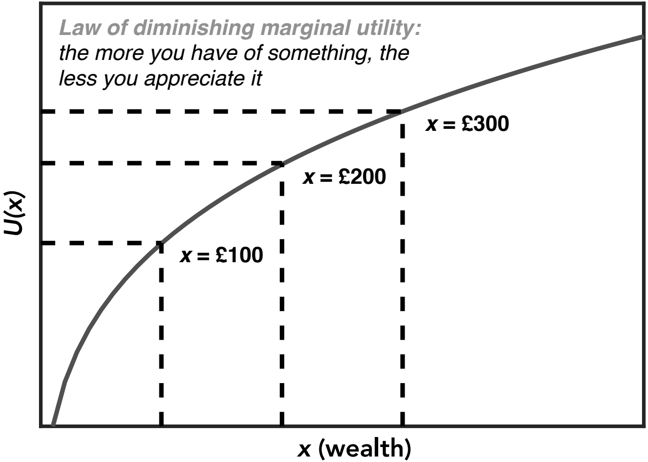
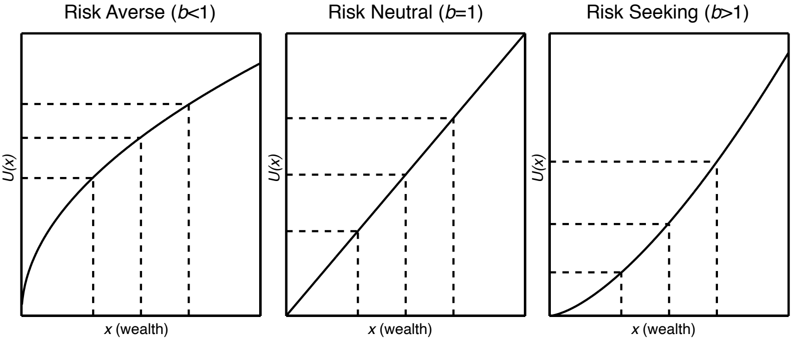
Risk Aversion



- The EUT utility function conforms to a power function of the form $u = x^b$, where $b < 1$

Risk Aversion, Risk Seeking & Risk Neutrality



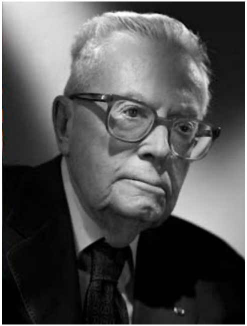
Performing Expected Utility Calculations

- When performing expected utility calculations, we transform monetary values into utilities using a utility function that captures the decision maker's risk preference
- Suppose we want to calculate the expected utilities of the prospects $\boldsymbol{q}$ = ($500) and $\boldsymbol{r}$ = ($1000, 0.5) for a risk averse decision maker
- We will set the value of b to 0.5 (reminder: $b < 1$ = risk averse)
- The utility of the former is calculated as $U(\boldsymbol{q}) = 500^{0.5}$ = **22.36**, whilst the latter is calculated as $U(\boldsymbol{r})$ = 0.5 $\times$ $1000^{0.5}$ = 15.81

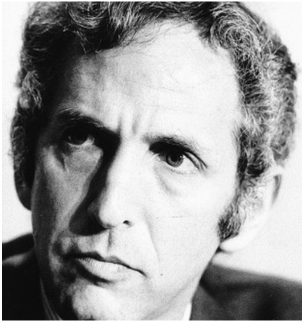
Performing Expected Utility Calculations

- Suppose you own £2 and are offered a gamble giving you a 50% chance of winning £1 and a 50% chance of losing £1
- If you are **risk averse**, what should you do?
  - $U(Accept)$ = (0.5 $\times$ $3^{0.5}$) + (0.5 $\times$ $1^{0.5}$) = 1.37
  - $U(Reject)$ = $2^{0.5}$ = **1.41**
- If you are **risk seeking**, what should you do?
  - $U(Accept)$ = (0.5 $\times$ $3^{1.5}$) + (0.5 $\times$ $1^{1.5}$) = **3.1**
  - $U(Reject)$ = $2^{1.5}$ = 2.83
- If you are **risk neutral**, what should you do?
  - $U(Accept)$ = (0.5 $\times$ 3^{1}) + (0.5 $\times$ 1^{1}) = 2
  - $U(Reject)$ = 2^{1} = 2

Violations of The Cancellation Axiom (Sure-Thing Principle)



Maurice Allais: winner of the Nobel Prize in Economics (1988)



Daniel Ellsberg: *The Most Dangerous Man in America*

The Allais Paradox (Allais, 1953)

Problem 1

You must choose between:

A: £500,000 for sure
B: £2,500,000 with probability 0.1, £500,000 with probability 0.89, nothing with probability 0.01

Problem 2

You must choose between:

C: £500,000 with probability 0.11, nothing with probability 0.89
D: £2,500,000 with probability 0.1, nothing with probability 0.89

The Allais Paradox (Allais, 1953)

Problem 1

You must choose between:

A: £500,000 for sure
B: £2,500,000 with probability 0.1, £500,000 with probability 0.89, nothing with probability 0.01

Problem 2

You must choose between:

C: £500,000 with probability 0.11, nothing with probability 0.89
D: £2,500,000 with probability 0.1, nothing with probability 0.89

Most people choose A in problem 1 and D in problem 2, but this pattern is inconsistent and violates the cancellation axiom

The Allais Paradox (Allais, 1953)

The preference for A over B implies:

$$U(£500,000) > 0.1\ U(£2,500,000) + 0.89\ U(£500,000) \tag{1}$$

But, equation 1 can be re-arranged by subtracting 0.89 U(£500,000) from both sides, so that:

$$U(£500,000) - 0.89\ U(£500,000) > 0.1\ U(£2,500,000)$$

which reduces to:

$$0.11\,U(£500,000) > 0.1\ U(£2,500,000) \tag{2}$$

Your preference for A over B implies you prefer a 0.11 chance of £500,000 to a 0.1 chance of £250,000,000

The Allais Paradox (Allais, 1953)

C: £500,000 with probability 0.11, nothing with probability 0.89
D: £2,500,000 with probability 0.1, nothing with probability 0.89

Here comes the rub! In problem 2, you preferred D over C (see above), which implies that:

$$0.1\,U(£2,500,000) > 0.11\;U(£500,000) \tag{3}$$

Which is the complete opposite to your preference in response to problem 1!

This is an example of a **preference reversal**

It is also an example of a violation of the cancellation axiom

Savage's (1954) Representation of The Allais Problem

Ticket number				
		1 (P = 0.01)	*2-11* (P = 0.1)	*12-100* (P = 0.89)
1	A	500,000	500,000	500,000
	B	0	2,500,000	500,000
2	C	500,000	500,000	0
	D	0	2,500,000	0

Savage's (1954) Representation of The Allais Problem

Ticket number				
		1 (P = 0.01)	*2-11* (P = 0.1)	*12-100* (P = 0.89)
1	A	500,000	500,000	
	B	0	2,500,000	
2	C	500,000	500,000	
	D	0	2,500,000	

When we eliminate (cancel) those states of the world that give the same outcome, it's clear that if you prefer A to B, you should also prefer C to D, on pain of inconsistency

The Ellsberg Paradox (Ellsberg, 1961)

Problem 1
Imagine an urn that contains 30 red balls, and 60 black or yellow balls in an unknown proportion. One ball is to be drawn at random. Would you bet on (1) Red or (2) Black?

Table: Payoffs for Ellsberg's Problem 1

Number of balls	*30*	*60*	
Colour of ball	Red	Black	Yellow
1: bet on Red	£100	£0	£0
2: bet on Black	£0	£100	£0

The Ellsberg Paradox (Ellsberg, 1961)

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2: bet on Black	£0	£100	£0

Most people prefer to bet on Red (option 1) in this case

The Ellsberg Paradox (Ellsberg, 1961)

Problem 2
Now you have to decide between (3) betting on 'Red or Yellow' or (4) betting on 'Black or Yellow'. Which would you choose?

Table: Payoffs for Ellsberg's Problem 2

Number of balls	*30*	*60*	
Colour of ball	Red	Black	Yellow
3: bet on Red or Yellow	£100	£0	£100
4: bet on Black or Yellow	£0	£100	£100

The Ellsberg Paradox (Ellsberg, 1961)

Problem 2

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Number of balls	*30*		*60*
Colour of ball	Red	Black	Yellow
3: bet on Red or Yellow	£100	£0	£100
4: bet on Black or Yellow	£0	£100	£100

Most people prefer to bet on Black or Yellow (option 4) in this case

But this is a violation of the cancellation axiom, since the two pairs of options differ only in their third component, which is constant for either pair

The Ellsberg Paradox (Ellsberg, 1961)

Table: Payoffs for Ellsberg's Problem 1

Number of balls	*30*	*60*	
Colour of ball	Red	Black	Yellow
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Table: Payoffs for Ellsberg's Problem 2

Number of balls	*30*	*60*	
Colour of ball	Red	Black	Yellow
3: bet on Red or Yellow	£100	£0	
4: bet on Black or Yellow	£0	£100	

Ambiguity Aversion

- The two rejected options (2 and 3) have something in common: the exact probability of winning is unclear—the probabilities are ambiguous
- By contrast the favoured options (1 and 4) are not associated with ambiguous probabilities
- The observed choices seem to reflect an unwillingness to take on gambles with ambiguous probabilities
- Ellsberg termed this phenomenon **ambiguity aversion**

Summary

- We introduced the standard economic model of decision making under risk and uncertainty (EUT)
- EUT is a normative model—it prescribes how people *should* make decisions if they are rational
- Assumes people's preferences satisfy certain rationality axioms and that they have specific preferences towards risk
- People's intuitive preferences often violate these rationality axioms (Allais' and Ellsberg's paradoxes)
- In the next lecture, we consider how people *actually* make decisions and introduce the most successful model of decision making—**prospect theory**