Three-Factor ANOVA

PSYC214: Statistics For Group Comparisons

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Week 9

Learning Objectives

- Procedures for analysing and interpreting three-factor ANOVA
- How to decompose a three-way interaction:
  - splitting the design and analysing it as a series of two-factor ANOVAs
- Examples:
  - $2 \times 2 \times 2$ fully within-participants ANOVA
  - $2 \times 2 \times 2$ mixed ANOVA
- General things to consider

Three-Factor ANOVA

- Three-factor ANOVAs are common in psychology
- In such designs, there are three possible two-way interactions:
  - A $\times$ B
  - A $\times$ C
  - B $\times$ C
- There is also the possibility of a three-way interaction:
  - A $\times$ B $\times$ C
- Complexity of interpreting these designs arises when the three-way interaction is significant

Three-Factor ANOVA

- Basic design principles of earlier lectures still apply
- A between-participants design is still relatively simple, with only a single error term for all effects
- However, a $2 \times 2 \times 2$ design would require at least 160 participants (obeying our maxim of $N = 20$ per cell)
- Problems with fully within-participants and mixed designs apply equally to three-factor designs
- Try to avoid exceeding two levels per factor where possible

Three-Factor ANOVA

- The most straightforward outcome is when the three-way interaction is not significant
- Where this occurs, one or more of the two-way interactions may be significant
- In which case, each significant two-way interaction should be investigated separately of the others
- The procedures for interpreting each interaction are the same as those discussed in previous lectures
- For example, if the A $\times$ B two-way interaction is significant, the simple main effects of factor A at B, and factor B at A can be investigated

Three-Factor ANOVA

- The simplest case arises when none of the interactions are significant
- In this case, the outcome must be interpreted in terms of the main effects, if any of these are significant
- If nothing is significant, then unless specific pairwise comparisons are planned, the analysis is complete

Dealing With A Significant Three-Way Interaction

- A significant three-way interaction occurs when there are different two-way interactions between two of the factors according to the levels of the third factor
- The simplest way to analyse a significant three-way interaction is to reanalyse it as a series of two-factor ANOVAs, e.g. :
  1. a 2 (factor A: level A_1 vs. level A_2) $\times$ 2 (factor B: level B_1 vs. level B_2) ANOVA at level C_1 of factor C
  2. a 2 (factor A: level A_1 vs. level A_2) $\times$ 2 (factor B: level B_1 vs. level B_2) ANOVA at level C_2 of factor C
- Any significant interactions would be followed up with a simple main effects analysis

Memory and Context: A 2 × 2 × 2 Fully Within-Participants Design

- A memory researcher wants to know if memory is better when material is tested in the same context it was learned in
- They also want to know whether recall and recognition memory are equally context dependent
- The researcher manipulates three factors in a 2 × 2 × 2 fully within-participants design:
  1. memory test (recall vs. recognition)
  2. learning context (learn under water vs. learn land)
  3. testing context (test under water vs. test land)
- Participants given words to remember in a learning context → memory for the words tested via recall or recognition
- Dependent measure is the number of words remembered correctly

Raw Data For Memory and Context Study

Table: A $2 \times 2 \times 2$ factorial design

Factor A: Task	*Level A_1 recall*				*Level A_2 recognition*			
Factor B: Learning	*Level B_1 under*		*Level B_2 land*		*Level B_1 under*		*Level B_2 land*	
Factor C: Testing	*C_1 under*	*C_2 land*	*C_1 under*	*C_2 land*	*C_1 under*	*C_2 land*	*C_1 under*	*C_2 land*
P_1	8	5	3	7	5	5	7	6
P_2	9	6	3	8	7	6	5	8
P_3	7	5	4	6	6	7	5	6
P_4	8	4	4	5	7	5	6	5
P_5	6	3	3	8	5	4	6	4

Aggregate Data For Memory and Context Study

Table: A 2 × 2 × 2 factorial design

	Level A_1 recall task				*Level A_2 recognition task*		
	Level B_1 under	*Level B_2 land*	*Overall*		*Level B_1 under*	*Level B_2 land*	*Overall*
Level C_1 *under water*	**7.6**	**3.4**	5.5	Level C_1 *under water*	**6**	**5.8**	5.9
Level C_2 *on land*	**4.6**	**6.8**	5.7	Level C_2 *on land*	**5.4**	**5.8**	5.6
Overall	6.1	5.1	5.6		5.7	5.8	5.8

ANOVA Table For Memory and Context Study

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (memory task)	0.225	1	0.225	1.000	0.374
Error $A \times P$	0.900	4	0.225		
B (learning context)	2.025	1	2.025	1.588	0.276
Error $B \times P$	5.100	4	1.275		
C (testing context)	0.025	1	0.025	0.014	0.911
Error $C \times P$	7.100	4	1.775		
$A \times B$	3.025	1	3.025	2.951	0.161
Error $A \times B \times P$	4.100	4	1.025		
$A \times C$	0.625	1	0.625	0.714	0.446
Error $A \times C \times P$	3.500	4	0.875		
$B \times C$	30.625	1	30.625	27.222	0.006
Error $B \times C \times P$	4.500	4	1.125		
$A \times B \times C$	21.025	1	21.025	27.129	0.006
Error $A \times B \times C \times P$	3.10	4	0.775		
P (participants)	10.900	4	2.733		

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P (participants)	10.900	4	2.733		

Interpreting The Significant Three-Way Interaction

- To decompose our significant three-way interaction, we first need to decide which factor to split our design by
- The obvious choice is factor A (memory task: recall vs. recognition)
- Next, we perform two two-factor ANOVAs:
  1. 2 (learning context: learn under water vs. learn land) $\times$ 2 (testing context: test under water vs. test land) ANOVA for the recall memory test condition only
  2. 2 (learning context: learn under water vs. learn land) $\times$ 2 (testing context: test under water vs. test land) ANOVA for the recognition memory test condition only

ANOVA Table For Recall Memory Task

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
B (learning context)	5.000	1	5.000	3.636	0.129
Error $B \times P$	5.500	4	1.375		
C (testing context)	0.200	1	0.200	0.186	0.688
Error $C \times P$	4.300	4	1.075		
$B \times C$	51.200	1	51.200	62.061	0.001
Error $B \times C \times P$	3.300	4	0.825		
P (participants)	5.300	4	1.333		

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P (participants)	5.300	4	1.333		

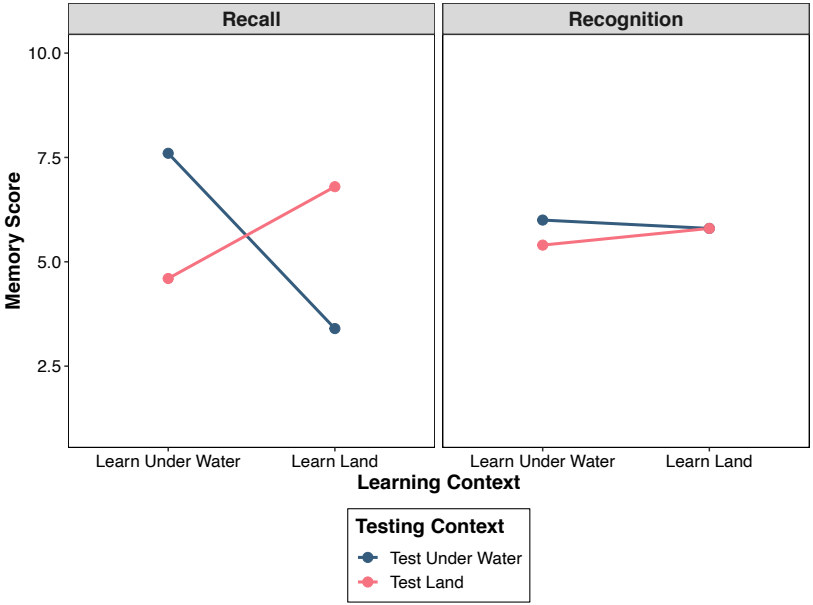
Simple Main Effects Table For Recall Memory Task

Source	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*P*
learning context at					
test under water	44.100	1	44.100	32.073	0.005
test land	12.100	1	12.100	8.800	0.041
Error term	5.50	4	1.375		
testing context at					
learn under water	22.500	1	22.500	20.930	0.010
learn land	28.900	1	28.900	26.884	0.007
Error term	4.300	4	1.075		

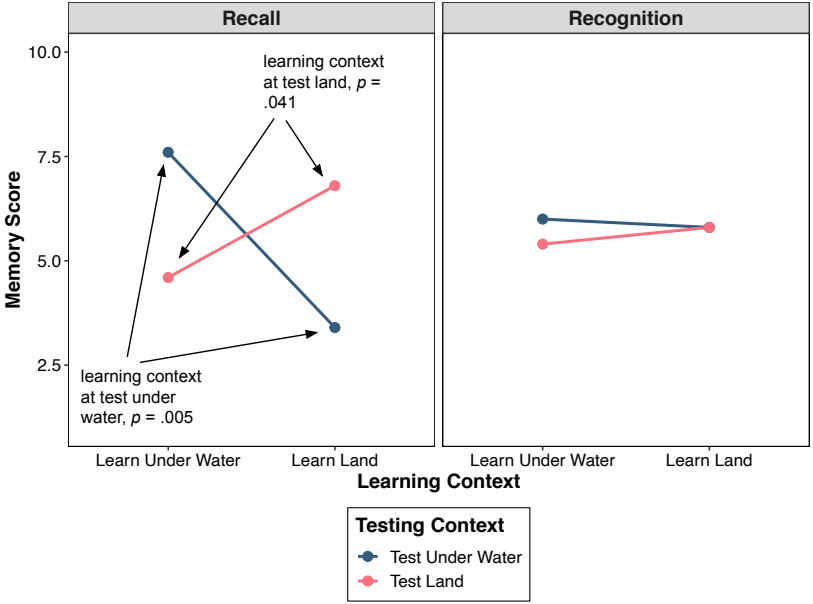
ANOVA Table For Recognition Memory Task

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
B (learning context)	0.050	1	0.050	0.054	0.828
Error $B \times P$	3.700	4	0.925		
C (testing context)	0.450	1	0.450	0.286	0.621
Error $C \times P$	6.300	4	1.575		
$B \times C$	0.450	1	0.450	0.419	0.553
Error $B \times C \times P$	4.300	4	1.075		
P (participants)	6.500	4	1.633		

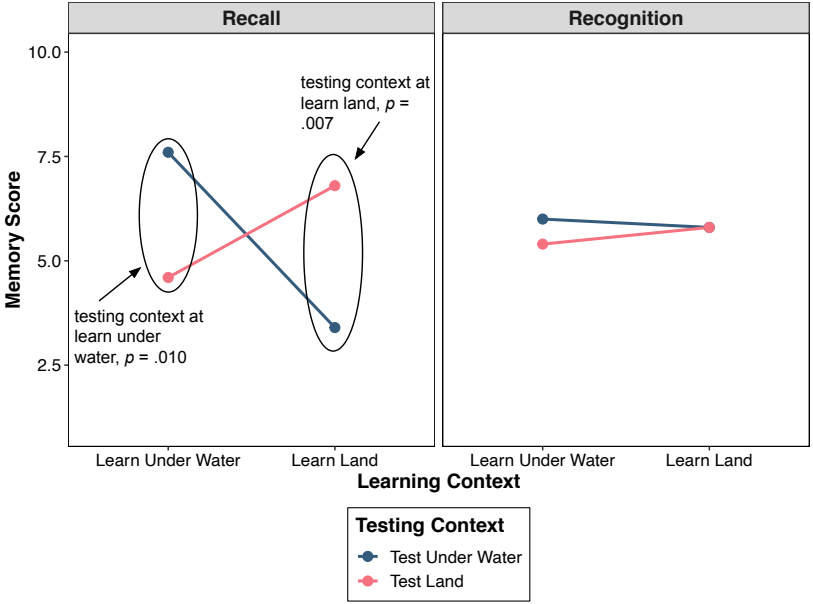
Interaction Plots For Memory and Context Study



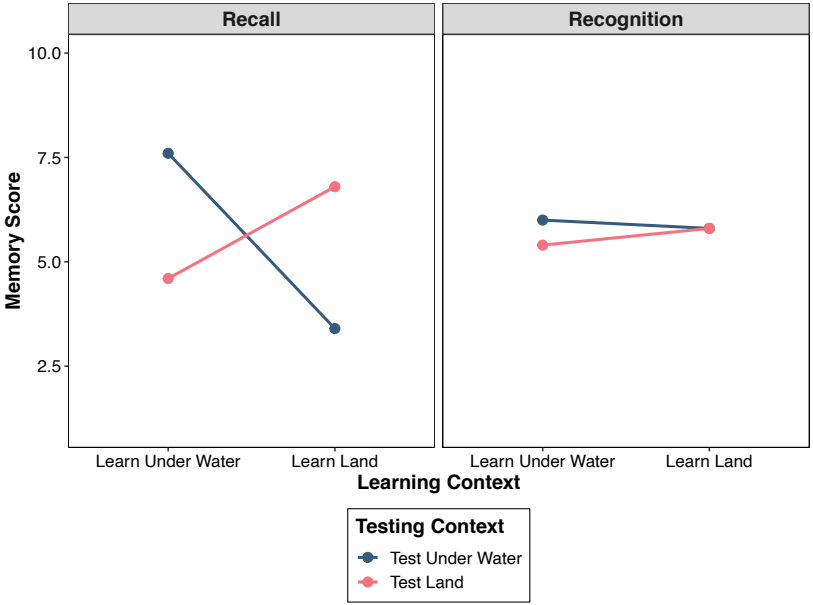
Interaction Plots For Memory and Context Study



Interaction Plots For Memory and Context Study



Interaction Plots For Memory and Context Study



Learning To Pronounce Irregular Words: A 2 × 2 × 2 Mixed Design

- A researcher wants to investigate the development in children's ability to pronounce regular and irregular words
- The researcher adopts a 2 × 2 × 2 mixed design:
  1. age (7 years old vs. 9 years old) is between-participants
  2. word frequency (low vs. high) is within-participants
  3. word type (regular vs. irregular) is within-participants
- Participants are given 10 words to pronounce in each category (40 words in total)
- Dependent measure is the number of pronunciation errors

Raw Data For Word Pronunciation Study

Table: A 2 × 2 × 2 factorial design

Factor A: Age	*Level A_1 7-years-old*					*Level A_2 9-years-old*			
Factor B: Frequency	*Level B_1 high*		*Level B_2 low*			*Level B_1 high*		*Level B_2 low*	
Factor C: Word type	*C_1 reg*	*C_2 irr*	*C_1 reg*	*C_2 irr*		*C_1 reg*	*C_2 irr*	*C_1 reg*	*C_2 irr*
P_1	6	7	5	6	P_6	4	4	3	6
P_2	7	5	6	7	P_7	3	4	4	7
P_3	5	6	7	6	P_8	4	3	5	9
P_4	6	7	5	7	P_9	5	5	3	8
P_5	6	6	5	7	P_{10}	3	4	3	7

Aggregate Data For Word Pronunciation Study

Table: A 2 × 2 × 2 factorial design

	Level A_1 7-years-old				*Level A_2 9-years-old*		
	Level B_1 high	*Level B_2 low*	*Overall*		*Level B_1 high*	*Level B_2 low*	*Overall*
Level C_1 *regular*	6.0	5.6	5.8	Level C_1 *regular*	3.8	3.6	3.7
Level C_2 *irregular*	6.2	6.6	6.4	Level C_2 *irregular*	4.0	7.4	5.7
Overall	6.1	6.1	6.1		3.9	5.5	4.7

ANOVA Table For Word Pronunciation Study

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
A (age)	19.600	1	19.600	34.844	< .001
Between error S/A	4.500	8	0.562		
B (frequency)	6.400	1	6.400	5.885	0.042
Error $B \times S/A$	8.700	8	1.087		
C (word type)	16.900	1	16.900	36.541	< .001
Error $C \times S/A$	3.700	8	0.462		
$A \times B$	6.400	1	6.400	5.885	0.042
Error $B \times S/A$	8.700	8	1.087		
$A \times C$	4.900	1	4.900	10.595	0.012
Error $C \times S/A$	3.700	8	0.462		
$B \times C$	12.100	1	12.100	17.600	0.003
Error $B \times C \times S/A$	5.500	8	0.688		
$A \times B \times C$	4.900	1	4.900	7.127	0.028
Error $B \times C \times S/A$	5.500	8	0.688		

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Interpreting The Significant Three-Way Interaction

- To decompose our significant three-way interaction, we first need to decide which factor to split our design by
- The obvious choice is our between-participants factor A (age: 7 year olds vs. 9 year olds)
- Next, we perform two two-factor ANOVAs:
  1. 2 (frequency: low vs. high) $\times$ 2 (word type: regular vs. irregular) ANOVA for the 7 year olds only
  2. 2 (frequency: low vs. high) $\times$ 2 (word type: regular vs. irregular) ANOVA for the 9 year olds only

ANOVA Table For 7 Year Olds

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
B (frequency)	0.000	1	0.000	0.000	1.000
Error $B \times P$	2.500	4	0.625		
C (word type)	1.800	1	1.800	5.885	0.178
Error $C \times P$	2.700	4	0.675		
$B \times C$	0.800	1	0.800	5.885	0.405
Error $B \times C \times P$	3.700	4	0.925		
P (participants)	0.300	4	0.075		

ANOVA Table For 9 Year Olds

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
B (frequency)	12.800	1	12.800	8.258	0.045
Error $B \times P$	6.200	4	1.550		
C (word type)	20.000	1	20.000	80.000	< .001
Error $C \times P$	1.000	4	0.250		
$B \times C$	16.200	1	16.200	36.000	0.004
Error $B \times C \times P$	1.800	4	0.450		
P (participants)	4.200	4	1.050		

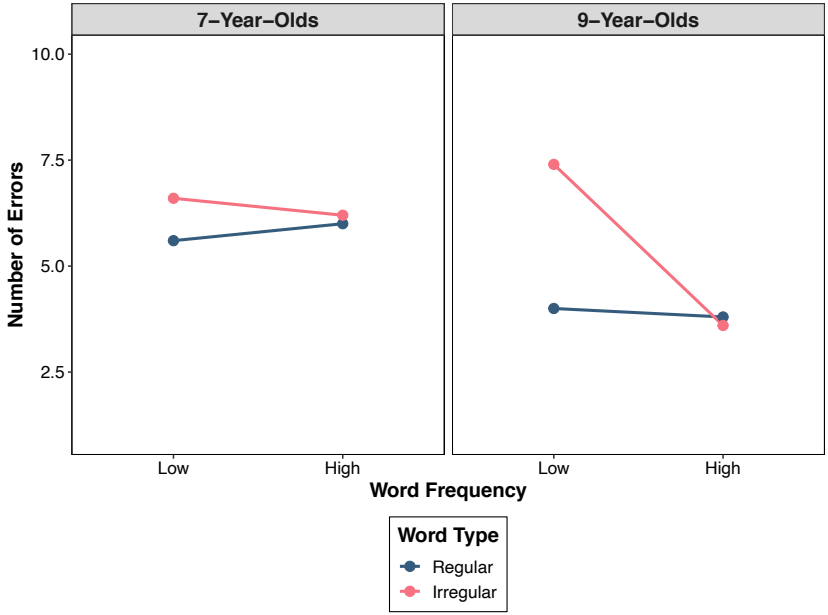
ANOVA Table For 9 Year Olds

Source	Sum of Squares	Degrees of Freedom	Mean Square	F	P
B (frequency)	12.800	1	12.800	8.258	0.045
Error $B \times P$	6.200	4	1.550		
C (word type)	20.000	1	20.000	80.000	< .001
Error $C \times P$	1.000	4	0.250		
$B \times C$	16.200	1	16.200	36.000	0.004
Error $B \times C \times P$	1.800	4	0.450		
P (participants)	4.200	4	1.050		

Simple Main Effects Table For 9 Year Olds

Source	Sum of Squares	Degrees of Freedom	Mean Square	*F*	*P*
word frequency at					
regular words	0.100	1	0.100	0.065	0.812
irregular words	28.900	1	28.900	18.645	0.013
Error term	6.200	4	1.550		
word type at					
low frequency	36.100	1	36.100	144.400	< .001
high frequency	0.100	1	0.100	0.400	0.561
Error term	1.000	4	0.250		

Interaction Plots For Word Pronunciation Study



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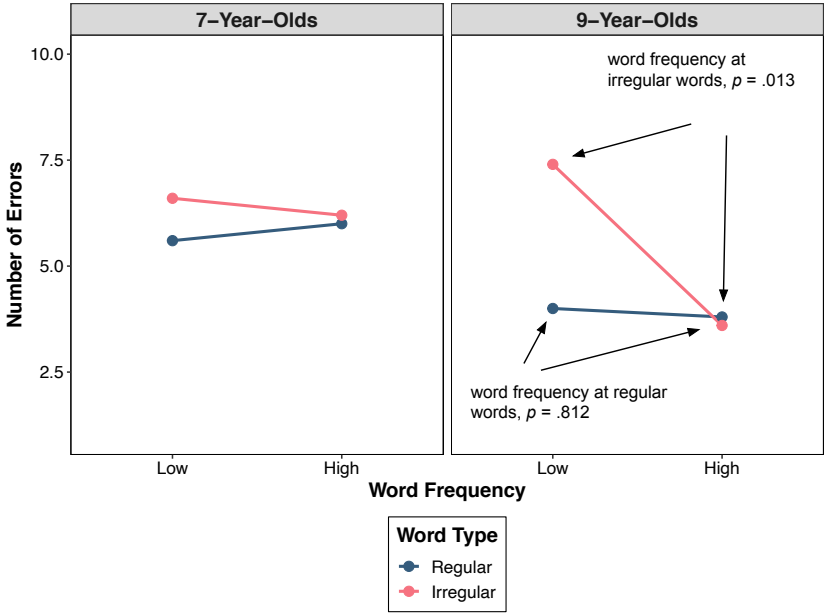

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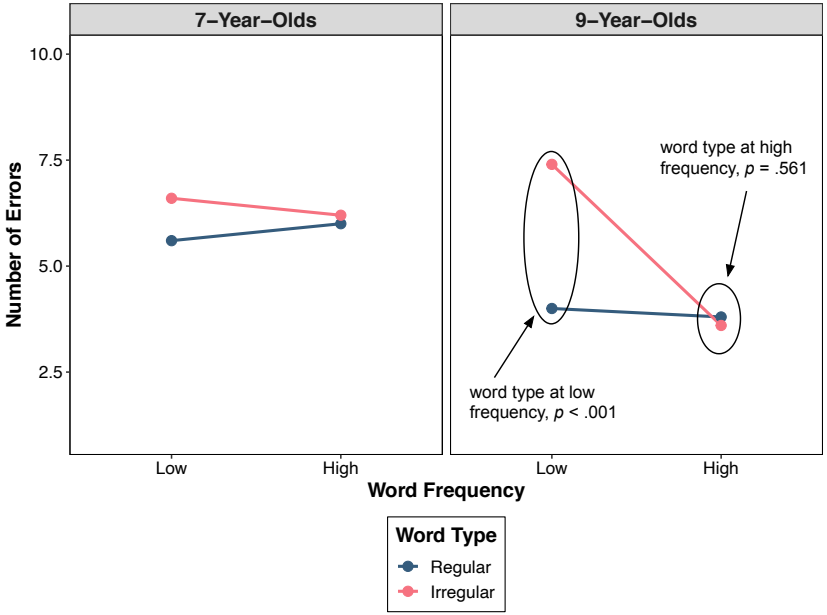

A Final Note On Interpreting Three-Way Interactions

- In both of these examples, one of the two-factor ANOVAs returned a significant interaction, whereas the other returned a non-significant interaction
- This will **not** always be the case
- Sometimes the interaction for each two-factor ANOVA will be significant and both will need to be followed up with a simple main effects analysis
- Under these conditions, the simple main effects for the two interactions will differ in direction and/or size of their trends

General Points

- As always, start at the bottom of the ANOVA table and work your way up
- If the three-way interaction is significant, then this must be analysed
- If not, then each of the significant two-way interactions should be analysed independently
- If none of the two-way interactions is significant, the ANOVA results may be described in terms of the main effects, with follow-up tests for any factors with three or more levels

In Next Week's Lab ...

- Running a three-factor (fully within-participants and mixed) ANOVA in R

References

Roberts, M. J., & Russo, R. (1999, Chapter 12). *A student's guide to Analysis of Variance*. Routledge: London.