# Two-Factor Between-Participants Designs 

PSYC214: Statistics For Group Comparisons

Mark Hurlstone
Lancaster University
Week 7

## Learning Objectives

- How to calculate $F$ ratios for two-factor between-participants designs
- How to calculate simple main effects, if the interaction is significant

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## Two-Factor Between-Participants Designs

- The simplest two-factor between-participants design is a $2 \times 2$ factorial design:
- there are two factors, each with two levels, yielding a total of four cells or conditions
- each participant contributes a single score to one condition only
- We can ask whether either of the main effects is significant
- We can also ask whether the interaction is significant

```
2 < 2 Factorial
```


## A Typical Between-Participants $2 \times 2$ Design

| $B_{1}$ | $\begin{aligned} & \mathrm{P}_{1} \mathrm{P}_{2} \mathrm{P}_{3} \\ & \mathrm{P}_{4} \mathrm{P}_{5} \mathrm{P}_{6} \\ & \mathrm{P}_{7} \mathrm{P}_{8} \mathrm{P}_{9} \end{aligned}$ | $\begin{aligned} & \mathrm{P}_{10} \mathrm{P}_{11} \mathrm{P}_{12} \\ & \mathrm{P}_{13} \mathrm{P}_{14} \mathrm{P}_{15} \\ & \mathrm{P}_{16} \mathrm{P}_{17} \mathrm{P}_{18} \end{aligned}$ | Mean $\mathrm{B}_{1}$ |
| :---: | :---: | :---: | :---: |
| $B_{2}$ | $\begin{aligned} & \mathrm{P}_{19} \mathrm{P}_{20} \mathrm{P}_{21} \\ & \mathrm{P}_{22} \mathrm{P}_{23} \mathrm{P}_{24} \\ & \mathrm{P}_{25} \mathrm{P}_{26} \mathrm{P}_{27} \end{aligned}$ | $\begin{aligned} & \mathrm{P}_{28} \mathrm{P}_{29} \mathrm{P}_{30} \\ & \mathrm{P}_{31} \mathrm{P}_{32} \mathrm{P}_{33} \\ & \mathrm{P}_{34} \mathrm{P}_{35} \mathrm{P}_{36} \end{aligned}$ | Mean $\mathrm{B}_{2}$ |

typical between-participants $2 \times 2$ design. Each participant only performs one of the four possible combinations of conditions

## Main Effects

| $\mathbf{A}_{\mathbf{1}}$ | $\boldsymbol{A}_{2}$ |
| :---: | :---: |
| $\mathrm{P}_{1} \mathrm{P}_{2} \mathrm{P}_{3}$ |  |
| $\mathrm{P}_{4} \mathrm{P}_{5} \mathrm{P}_{6}$ |  |
| $\mathrm{P}_{7} \mathrm{P}_{8} \mathrm{P}_{9}$ |  |
| $\mathrm{P}_{19} \mathrm{P}_{20} \mathrm{P}_{21}$ |  |
| $\mathrm{P}_{22} \mathrm{P}_{23} \mathrm{P}_{24}$ |  |
| $\mathrm{P}_{25} \mathrm{P}_{26} \mathrm{P}_{27}$ |  |
| Mean $\mathrm{A}_{1}$ | $\mathrm{P}_{10} \mathrm{P}_{11} \mathrm{P}_{12}$ <br> $\mathrm{P}_{13} \mathrm{P}_{14} \mathrm{P}_{15}$ <br> $\mathrm{P}_{16} \mathrm{P}_{17} \mathrm{P}_{18}$ <br> $\mathrm{P}_{28} \mathrm{P}_{29} \mathrm{P}_{30}$ <br> $\mathrm{P}_{31} \mathrm{P}_{32} \mathrm{P}_{33}$ <br> $\mathrm{P}_{34} \mathrm{P}_{35} \mathrm{P}_{36}$ |
| Mean $\mathrm{A}_{2}$ |  |

Main effect of $A$ : Is the difference between means of $A_{1}$ and $A_{2}$ significant (ignoring factor B )?

$$
\begin{aligned}
& \mathbf{B 1}_{1} \begin{array}{ll}
\mathrm{P}_{1} \mathrm{P}_{2} \mathrm{P}_{3} & \mathrm{P}_{10} \mathrm{P}_{11} \mathrm{P}_{12} \\
\mathrm{P}_{4} \mathrm{P}_{5} \mathrm{P}_{6} & \mathrm{P}_{13} \mathrm{P}_{14} \mathrm{P}_{15} \\
\mathrm{P}_{7} \mathrm{P}_{8} \mathrm{P}_{9} & \mathrm{P}_{16} \mathrm{P}_{17} \mathrm{P}_{18}
\end{array} \quad \begin{array}{c}
\text { Mean } \\
\mathrm{B}_{1}
\end{array} \\
& \mathbf{B 2}_{2} \begin{array}{lll}
\mathrm{P}_{19} \mathrm{P}_{20} \mathrm{P}_{21} & \mathrm{P}_{28} \mathrm{P}_{29} \mathrm{P}_{30} \\
\mathrm{P}_{22} \mathrm{P}_{23} \mathrm{P}_{24} & \mathrm{P}_{31} \mathrm{P}_{32} \mathrm{P}_{33} \\
\mathrm{P}_{25} \mathrm{P}_{26} \mathrm{P}_{27} & \mathrm{P}_{34} \mathrm{P}_{35} \mathrm{P}_{36} & \text { Mean } \\
\mathrm{B}_{2}
\end{array}
\end{aligned}
$$

Main effect of B : Is the difference between means of $B_{1}$ and $B_{2}$ significant (ignoring factor A )?
$2 \times 2$ Factorial Design

Main Effects
Simple Main Effects
Analysis a 2 2 Design

Basic Ratios
SS WITHIN, BETWEEN, $\&$ TOTAL
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## Simple Main Effects of Factor A

$B_{1}$


Mean $A_{1} \quad$ Mean $A_{2}$
（at $B_{1}$ ）
（at $\mathrm{B}_{1}$ ）

B2 $\begin{aligned} & \mathrm{P}_{19} \mathrm{P}_{20} \mathrm{P}_{21} \\ & \mathrm{P}_{22} \mathrm{P}_{23} \mathrm{P}_{24} \\ & \mathrm{P}_{25} \mathrm{P}_{26} \mathrm{P}_{27}\end{aligned}$
Mean $\mathrm{A}_{1}$
（at $B_{2}$ ）

$$
\begin{gathered}
\hline \mathrm{P}_{28} \mathrm{P}_{29} \mathrm{P}_{30} \\
\mathrm{P}_{31} \mathrm{P}_{32} \mathrm{P}_{33} \\
\mathrm{P}_{34} \mathrm{P}_{35} \mathrm{P}_{36} \\
\hline \text { Mean } \mathrm{A}_{2} \\
\text { (at } \mathrm{B}_{2} \text { ) }
\end{gathered}
$$

Simple main effect of $A$ at $B_{1}$ ： Is the difference between means of $A_{1}$ and $A_{2}$ significant at $B_{1}$ of factor $B$ ？

Simple main effect of $A$ at $B_{2}$ ：
Is the difference between means of $A_{1}$ and $A_{2}$ significant at $B_{2}$ of factor $B$ ？

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ss interacion


ANoVA Table
Simple Main Effects

## Simple Main Effects of Factor B

$$
\begin{aligned}
& \mathbf{A}_{1} \\
& \text { B1 }^{\begin{array}{l}
P_{1} P_{2} P_{3} \\
P_{4} P_{5} P_{6} \\
P_{7} P_{8} P_{9}
\end{array}} \begin{array}{c}
\text { Mean } B_{1} \\
\text { (at } \left.A_{1}\right)
\end{array} \\
& B_{2} \begin{array}{|c|c}
\begin{array}{l}
\mathrm{P}_{19} \mathrm{P}_{20} \mathrm{P}_{21} \\
\mathrm{P}_{22} \mathrm{P}_{23} \mathrm{P}_{24} \\
\mathrm{P}_{25} \mathrm{P}_{26} \mathrm{P}_{27} \\
\hline
\end{array} & \begin{array}{c}
\text { Mean } \mathrm{B}_{2} \\
\text { (at } \left.\mathrm{A}_{1}\right)
\end{array} \\
\hline
\end{array}
\end{aligned}
$$

Simple main effect of $B$ at $A_{1}$ : Is the difference between means of $B_{1}$ and $B_{2}$ significant at $A_{1}$ of factor $A$ ?


$$
\begin{aligned}
& \begin{array}{l}
\mathrm{P}_{10} \mathrm{P}_{11} \mathrm{P}_{12} \\
\mathrm{P}_{13} \mathrm{P}_{14} \mathrm{P}_{15} \\
\mathrm{P}_{16} \mathrm{P}_{17} \mathrm{P}_{18}
\end{array} \quad \text { Mean } \mathrm{B}_{1} \\
& \text { (at } \left.\mathrm{A}_{2}\right)
\end{aligned}
$$

| $\mathrm{P}_{28} \mathrm{P}_{29} \mathrm{P}_{30}$ | Mean $\mathrm{B}_{2}$ |
| :---: | :---: |
| $\mathrm{P}_{31} \mathrm{P}_{32} \mathrm{P}_{33}$ | $\left(\right.$ at $\left.\mathrm{A}_{2}\right)$ |
| $\mathrm{P}_{34} \mathrm{P}_{35} \mathrm{P}_{36}$ |  |

Simple main effect of $B$ at $A_{\mathbf{2}}$ : Is the difference between means of $B_{1}$ and $B_{2}$ significant at $A_{2}$ of factor $A$ ?

## Simple Main Effects

－There are two ways a pair of simple main effects may differ in their trends：
（1）one of a pair has a significant difference but not the other．For example， the mean of $A_{1}$ differs from the mean of $A_{2}$ at level $B_{2}$ but not at level $B_{1}$
（2）both simple main effects are significant，but in the opposite direction．For example，the mean of $A_{1}$ is greater than the mean of $A_{2}$ at level $B_{1}$ ，but the pattern is reversed at level $B_{2}$

## Simple Main Effects

Factor A
Factor A

- Level ( $\mathrm{A}_{1}$ )
- Level ( $\mathrm{A}_{1}$ )
- Level $\left(\mathrm{A}_{2}\right)$
- Level $\left(\mathrm{A}_{2}\right)$
m.hurlstone@


## Analysis a $2 \times 2$ Between-Participants Factorial Design

- The first stage of analysis seeks to uncover which of the two main effects and interactions are significant
- If the interaction is significant, then in a second stage we perform a simple main effects analysis
- Although a second factor has been added, the $F$ ratio remains the same:

$$
F=\frac{\text { treatment effects }+ \text { experimental error }}{\text { experimental error }}
$$

- As this is a between-participants design:

$$
F=\frac{\text { between-group variance }}{\text { within-group variance }}
$$

Analysis a $2 \times$ 2 Design

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## Analysis a $2 \times 2$ Between-Participants Factorial Design

- The main difference is that there are now three $F$ ratios, one for each of the three effects

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SS Main Ellecis ss miteracion ANoVA Table

Simple Main Effects

## Hypothetical Data For COVID-19 Study

|  | Factor A: Fear |  |  |
| :--- | :--- | :--- | :--- |
| Factor $B:$ Level $A_{1}$ Level $A_{2}$ <br> Efficacy no fear appeal fear appeal | $P_{1}$ no efficacy message | $P_{1}$ | 5 |
|  | $P_{2}$ | 4 | $P_{13}$ |

## Hypothetical Data For COVID-19 Study

Factor A: Fear

|  |  | Level $A_{1}$ <br> no fear appeal | Level $A_{2}$ <br> fear appeal |  |
| :--- | :--- | :---: | :---: | :---: |
|  |  | Overall |  |  |
| Factor $B:$ | Level $B_{1}$ no efficacy message | 5.00 | 5.00 | 5.00 |
| Efficacy | Level $B_{2}$ efficacy message | 5.17 | 8.17 | 6.67 |
| Overall | 5.08 | 6.58 | 5.83 |  |

Main Effects
Simple Main Elfecis
Analysis a 2 2 Design

Data
Basic Ratios
SS WITHIN, BETWEEN, $\varepsilon$ TOTAL
SS Main Effects
SS Interaction DF
ANOVA Table
Simple Main Effects
Between-Group SS \& DF Simple Main Effects Tabl

## Notation

$$
\begin{gathered}
S S_{B E T W E E N}=\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}}-\frac{\left(\sum Y\right)^{2}}{N} \\
S S_{W I T H I N}=\sum Y^{2}-\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}} \\
S S_{T O T A L}=\sum Y^{2}-\frac{\left(\sum Y\right)^{2}}{N}
\end{gathered}
$$

## Notation

$$
\begin{gathered}
S S_{\text {BETWEEN }}=\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}}-\frac{\left(\sum Y\right)^{2}}{N} \\
S S_{W I T H I N}=\sum Y^{2}-\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}} \\
S S_{\text {TOTAL }}=\sum Y^{2}-\frac{\left(\sum Y\right)^{2}}{N}
\end{gathered}
$$

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Simple Main Eliecis
Analysis a 2 2 Design

Basic Ratios
S WITHIN, BETWEEN, \& TOTAL
SS Main Effects
SS Interaction
$\qquad$
Simple Main Effects

Between-Group SS \& DF

## Notation

$$
\begin{gathered}
S S_{\text {BETWEEN }}=\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}}-\frac{\left(\sum Y\right)^{2}}{N} \\
S S_{W I T H I N}=\sum Y^{2}-\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}} \\
S S_{\text {TOTAL }}=\sum Y^{2}-\frac{\left(\sum Y\right)^{2}}{N}
\end{gathered}
$$

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m.hurlstone@ lancaster.ac.uk

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Basic Ratios
SS WITHIN, BETWEEN, \& TOTAL

SS Main Effects
SS Interaction
ANOVA Table
Simple Main
Effects
Between-Group SS \& DF Simple Main Effects Table

## Notation

$$
\begin{gathered}
S S_{B E T W E E N}=\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}}-\frac{\left(\sum Y\right)^{2}}{N} \\
S S_{\text {WITHIN }}=\sum Y^{2}-\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}} \\
S S_{\text {TOTAL }}=\sum Y^{2}-\frac{\left(\sum Y\right)^{2}}{N}
\end{gathered}
$$

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Basic Ratios
SS WITHIN, BETWEEN, \& TOTAL
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SS Interaction

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Simple Main Effects Table

## Notation

$$
\frac{\left(\sum Y\right)^{2}}{N} \text { is } \frac{(\text { grand total })^{2}}{\text { the number of scores that make up the grand total }}
$$

$$
\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}} \text { is } \frac{\left(\text { level total of } A_{1}\right)^{2}+\left(\text { level total of } A_{2}\right)^{2}}{\text { the number of scores that make up each level }}
$$

$$
\sum Y^{2} \text { is } \frac{\left(\text { score }_{1}\right)^{2}+\left(\text { score }_{2}\right)^{2}+\left(\text { score }_{3}\right)^{2}(\text { and so on })}{1 \text { (only one number makes up each individual score })}
$$

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## Basic Ratios

$[T]$ : basic ratio of the grand total, $\frac{\left(\sum Y\right)^{2}}{N}$
$[A]$ : basic ratio of the level totals, $\frac{\left(\sum A_{1}\right)^{2}+\left(\sum A_{2}\right)^{2}}{N_{A}}$
$[Y]$ : basic ratio of the individual scores, $\sum Y^{2}$

## Basic Ratios

- To compute the components of a factorial between-participants ANOVA, two additional ratios are required
- $[B]$ is the basic ratio of the level totals of factor B. If there are two levels in factor $B$, then $[B]=$
$\frac{\left(\text { level total of } B_{1}\right)^{2}+\left(\text { level total of } B_{2}\right)^{2}}{\text { the number of scores that make up each level }}=\frac{\left(\sum B_{1}\right)^{2}+\left(\sum B_{2}\right)^{2}}{N_{B}}$
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Analysis a 2 2 Design

Basic Ratios
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SS Main Effects
SS interaction
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Simple Main Effects

## Basic Ratios

－$[A B]$ is the basic ratio of the cell totals，where a cell total is the total of all the scores in any one of the cells．For a $2 \times 2$ design，$[A B]=$

$$
\left(\begin{array}{l}
\text { cell total of } \left.A_{1} B_{1}\right)^{2}+\left(\text { cell total of } A_{1} B_{2}\right)^{2}+\left(\text { cell total of } A_{2} B_{1}\right)^{2}+\left(\text { cell total of } A_{2} B_{2}\right)^{2} \\
\hline
\end{array}\right.
$$

the number of scores in each cell

$$
=\left(\sum A_{1} B_{1}\right)^{2}+\left(\sum A_{1} B_{2}\right)^{2}+\left(\sum A_{2} B_{1}\right)^{2}+\left(\sum A_{2} B_{2}\right)^{2}
$$

$2 \times 2$ Factorial Design
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## Basic Ratios

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Simple Main Effects

## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



## Calculating Basic Ratios For The Hypothetical Data



- Within-group variance is a measure of the extent to which people within each of the groups behave differently, despite being treated alike
- For a $2 \times 2$ between-participants design, people have been treated exactly alike only within each of the four cells
- To calculate the error term, we compute and combine the Sums of Squares and degrees of freedom using the smallest unit of identically treated participants-the four cells
- This gives a single measure of experimental error that can be used for calculating the Fs for all the effects


## Calculating The Sum of Squares For The Error Term

- We calculate the error term, $S S_{\text {WITHin }}$, as follows:

$$
S S_{\text {WITHIN }}=[Y]-[A B] \quad S S_{\text {WITHIN }} \text { will be designated } S S_{S / A B}
$$

- This produces the error term that will be used to calculate all the Fs
- This is the overall measure of the extent to which participants behaved differently despite being treated alike


## Between-Group Sum of Squares

- We also need to calculate the total between-group Sum of Squares for the four cells
- This is a measure of the variability due to the various experimental treatments
- It is a measure of how distant each of the four cell means is from the grand mean
- It tells us the overall extent to which the treatments caused scores to differ
- The between-group Sum of Squares is calculated as:

$$
S S_{B E T W E E N}=[A B]-[T] \quad S S_{B E T W E E N} \text { will be designated } S S_{A B}
$$

$2 \times 2$ Factorial Design

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SS WITHIN, BETWEEN, \& TOTAL
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## Total Sum of Squares

－We also need to calculate the total Sum of Squares
－This is a measure of total variability for the entire data set irrespective of experimental treatments
－It is calculated as：

$$
S S_{T O T A L}=[Y]-[T]
$$

## Calculating The Sums of Squares For The Two Main Effects

- Two between-group sums of squares are required, one for each of the main effects
- Each main effect is treated as being completely independent from the other
- e.g., when calculating the main effect of factor A , the fact participants were treated in different ways at factor $B$ is ignored
- The Sums of Squares for the two main effects are calculated as:
for the between-group sums of squares for factor $A, S S_{A}=[A]-[T]$ for the between-group sums of squares for factor $B, S S_{B}=[B]-[T]$
$2 \times 2$ Factorial Design
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SS WITHIN, BETWEEN,
SS Main Effects

## Calculating The Sums of Squares For The Two Main Effects

- To test the significance of the interaction, a final Sums of Squares is required
- This is calculated as:

$$
S S_{\text {INTERACTION }}, S S_{A \times B}=[A B]-[A]-[B]+[T]
$$

- This is the variability in thee group means not accounted for by the main effects
- It is the variability caused by the interaction between factor A and factor B


## Calculating The Sums of Squares Discussed So Far

Within－group Sum of Squares：$S S_{S / A B}=[Y]-[A B]$

$$
=910-860.3333=49.67
$$

Total between－group Sum of Squares：$S S_{A B}=[A B]-[T]$

$$
=860.3333-816.6667=43.67
$$

Total Sum of Squares：$S S_{\text {TOTAL }}=[Y]-[T]$

$$
=910-816.6667=93.33
$$

## Calculating The Sums of Squares Discussed So Far

Between－group Sum of Squares for factor A：$S S_{A}=[A]-[T]$

$$
=830.1667-816.667=13.50
$$

Between－group Sum of Squares for factor B：$S S_{B}=[B]-[T]$

$$
=833.3333-816.6667=16.67
$$

Sum of Squares for interaction：$S S_{A \times B}=[A B]-[A]-[B]+[T]$

$$
=860.3333-830.1667-833.3333+816.6667=13.50
$$

## Degrees of Freedom

－For the main effects：

$$
\begin{array}{r}
d f_{A}=(\text { number of levels in factor } A-1)=(a-1) \\
(a \text { is the number of levels in factor } A)
\end{array} \begin{array}{r}
d f_{B}=(\text { number of levels in factor } B-1)=(b-1) \\
(b \text { is the number of levels in factor } B)
\end{array}
$$

－For the interaction：

$$
d f_{A \times B}=d f_{A} \times d f_{B}=(a-1)(b-1)
$$

Simple Main Effects
Analysis a 2 2 Design Data Basic Ratios SS WITHIN，BETWEEN，
TOTAL

## Degrees of Freedom

－For the within－group variance（the error term）：

$$
\begin{array}{r}
d f_{S / A B}=[(\text { number of cells }) \times(\text { number of scores in cell }-1)] \\
=a b(s-1)
\end{array}
$$

（ $s$ is the number of scores in a cell）
－For the total degrees of freedom：

$$
d f_{\text {TOTAL }}=(\text { total number of scores }-1)=(a b s)-1
$$

## Degrees of Freedom

- The various degrees of freedom should add up so that:

$$
d f_{T O T A L}=d f_{A}+d f_{B}+d f_{A \times B}+d f_{S / A B}
$$

## Calculating The Degrees of Freedom Discussed So Far

$$
\begin{gathered}
d f_{A}=(a-1)=2-1=1 \text { (factor } A \text { has two levels) } \\
d f_{B}=(b-1)=2-1=1 \text { (factor } B \text { has two levels) } \\
d f_{A \times B}=(a-1)(b-1)=1 \times 1=1 \\
d f_{S / A B}=a b(s-1)=2 \times 2(6-1)=20(\text { six participants per cell }) \\
d f_{T O T A L}=(a b s)-1=(2 \times 2 \times 6)-1=23
\end{gathered}
$$

## Summary ANOVA Table By Components

| Source | Sum of Squares | Degrees of freedom | Mean Square | F | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| A | $[A]-[T]$ | $(a-1)$ | $\frac{[A]-[T]}{(a-I)}$ | Mean Square ${ }_{A}$ <br> Mean Square ${ }_{S A B}$ | tables |
| B | $[B]-[T]$ | $(b-1)$ | $\frac{[B]-[T]}{(b-I)}$ | $\frac{\text { Mean Square }_{B}}{\text { Mean Square }_{S A B}}$ | tables |
| $A \times B$ | $\begin{aligned} & {[A B]-[A]} \\ & -[B]+[T] \end{aligned}$ | $(a-1)(b-1)$ | $\frac{[A B]-[A]-[B]+[T]}{(a-I)(b-I)}$ | $\frac{\text { Mean Square }_{A \times B}}{\text { Mean Square }}$ | tables |
| S/AB | $[Y]-[A B]$ | $a b(s-1)$ | $\frac{[Y]-[A B]}{a b(s-I)}$ |  |  |
| TOTAL | $[Y]-[T]$ | (abs) - I |  |  |  |

## ANOVA Table For Hypothetical Data

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | 13.50 | 1 |  |  |  |
| $B$ | 16.67 | 1 |  |  |  |
| $A \times B$ | 13.50 | 1 |  |  |  |
| S/AB | 49.67 | 20 |  |  |  |
| TOTAL | 93.33 | 23 |  |  |  |

Simple Main Effecis
Analysis a 2 2 Design


Basic Ratios
SS WITHIN, BETWEEN, \& TOTAL
SS Main Effects
SS interaction

ANOVA Table
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## ANOVA Table For Hypothetical Data

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | 13.50 | 1 | 13.50 |  |  |
| $B$ | 16.67 | 1 | 16.67 |  |  |
| $A \times B$ | 13.50 | 1 | 13.50 |  |  |
| S/AB | 49.67 | 20 | 2.48 |  |  |
| TOTAL | 93.33 | 23 | 4.06 |  |  |

## ANOVA Table For Hypothetical Data

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | 13.50 | 1 | 13.50 | 5.44 |  |
| $B$ | 16.67 | 1 | 16.67 | 6.72 |  |
| $A \times B$ | 13.50 | 1 | 13.50 | 5.44 |  |
| S／AB | 49.67 | 20 | 2.48 |  |  |
| TOTAL | 93.33 | 23 | 4.06 |  |  |

Simple Main Effects
Analysis a 2 2 Design

Basic Ratios SS WITHIN，BETWEEN，\＆ TOTAL
SS Main Effects
SS Interaction

ANOVA Table
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Between－Group SS \＆DF
Simple Main Effects Table

## ANOVA Table For Hypothetical Data

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | 13.50 | 1 | 13.50 | 5.44 | $<.05$ |
| $B$ | 16.67 | 1 | 16.67 | 6.72 | $<.05$ |
| $A \times B$ | 13.50 | 1 | 13.50 | 5.44 | $<.05$ |
| S/AB | 49.67 | 20 | 2.48 |  |  |
| TOTAL | 93.33 | 23 | 4.06 |  |  |

## ANOVA Table For Hypothetical Data

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | 13.50 | 1 | 13.50 | 5.44 | $<.05$ |
| $B$ | 16.67 | 1 | 16.67 | 6.72 | $<.05$ |
| $A \times B$ | 13.50 | 1 | 13.50 | 5.44 | $<.05$ |
| S/AB | 49.67 | 20 | 2.48 |  |  |
| TOTAL | 93.33 | 23 | 4.06 |  |  |

## Interaction Plot


m.huristone@ lancaster.ac.uk
$2 \times 2$ Factorial Design
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ANOVA Table
Simple Main Effects
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## Simple Main Effects

- If the interaction is significant, then we interpret it by analysing the simple main effects
- In a $2 \times 2$ design, these are simply pairwise comparisons, analogous to using four $t$-tests
- This involves calculating the between-group variance for each simple main effect, before dividing each variance by the error term $(S / A B)$ from the original ANOVA
- Thus, the significance of the simple main effects is evaluated using the
$2 \times 2$ Factorial Design
Sirucure
Main Elfects
Simple Main Effects

Analysis a 2 2 Design

Basic Ralios
SS WITHIN, BETWEEN TOTAL
SS Main Eliects SS Interaction DF

Simple Main Effects same error term used to test the significance of the main effects and interaction

## Simple Main Effects

Main effect of A: To find out whether the main effect of A is significant, calculate the between-group variance of the means of $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ in relation to the grand mean (ignoring factor B ). The bigger thee variance, the bigger the difference between thee means and the more likely that the difference is significant.

$2 \times 2$ Factorial Design

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Effects

## Simple Main Effects

| $B_{1}$ | $\begin{array}{r} \boldsymbol{A}_{1} \\ \mathrm{P}_{1} \mathrm{P}_{2} \mathrm{P}_{3} \\ \mathrm{P}_{4} \mathrm{P}_{5} \mathrm{P}_{6} \\ \mathrm{P}_{7} \mathrm{P}_{8} \mathrm{P}_{9} \\ \hline \text { Mean } \mathrm{A}_{1} \\ \text { (at } \mathrm{B}_{1} \text { ) } \\ \text { Rea } \\ \text { Mea } \\ \text { the } \end{array}$ | $\begin{aligned} & \boldsymbol{A}_{2} \\ & \mathrm{P}_{10} \mathrm{P}_{11} \mathrm{P}_{12} \\ & \mathrm{P}_{13} \mathrm{P}_{14} \mathrm{P}_{15} \\ & \mathrm{P}_{16} \mathrm{P}_{17} \mathrm{P}_{18} \\ & \begin{array}{c} \text { Mean } \mathrm{A}_{2} \\ \text { (at } \mathrm{B}_{1} \text { ) } \end{array} \\ & \text { ill of } \\ & \text { ores } \end{aligned}$ | Simple main effect of $A$ at $B_{1}$ : To find out whether this simple main effect is significant, calculate the between-group variance for means $A_{1}$ and $A_{2}$ in relation to the overall mean of all the $B_{1}$ scores. The bigger the variance, the bigger the difference between the means and the more likely that the difference is significant. |
| :---: | :---: | :---: | :---: |
| $B_{2}$ | $\begin{gathered} \mathrm{P}_{19} \mathrm{P}_{20} \mathrm{P}_{21} \\ \mathrm{P}_{22} \mathrm{P}_{23} \mathrm{P}_{24} \\ \mathrm{P}_{25} \mathrm{P}_{26} \mathrm{P}_{27} \\ \hline \begin{array}{c} \text { Mean } \mathrm{A}_{1} \\ \text { (at } \mathrm{B}_{2} \text { ) } \end{array} \end{gathered}$ | $\begin{gathered} \mathrm{P}_{28} \mathrm{P}_{29} \mathrm{P}_{30} \\ \mathrm{P}_{31} \mathrm{P}_{32} \mathrm{P}_{33} \\ \mathrm{P}_{34} \mathrm{P}_{35} \mathrm{P}_{36} \\ \hline \text { Mean } \mathrm{A}_{2} \\ \text { (at } \mathrm{B}_{2} \text { ) } \end{gathered}$ | Ignore $B_{2}$ when testing the significance of the simple main effect of $A$ at $B_{1}$ |

$2 \times 2$ Factorial Design
Structure
Main Etects
Simple Main Eliecis
Analysis a 2 2 Design Data
Basic Railos SS within, between, \& TOTAL
SS Main Elfects
SS Interacion
DF
ANOVA Table
Simple Main
Effects
Beween-Group SS \& DF Simple Main Effects Table

## Calculating Between－Group Sum of Squares

－The formula for calculating a between－group Sum of Squares is the basic ratio of the group totals of interest，minus the basic ratio of the total of these totals［ 7 ］
－For example，the formula for calculating the between－group variance for the main effect of factor A is $[A]-[T]$
－The basic ratios used to calculate the between－group variances for the simple main effects are analogous to these
$2 \times 2$ Factorial Design
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Analysis a 2 2 Design

Basic Railios
Ss within，between TOTAL
SS Man Elilects
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anova tede
Simple Main Effects
Between－Group SS \＆DF

## Calculating Between-Group Sum of Squares

- For example:
- $\left[A_{B_{1}}\right]$ is the basic ratio of factor $A$, but only for the $B_{1}$ scores: square the total for $A_{1} B_{1}$, square the total for $A_{2} B_{1}$, add the squares together and divide by the
$2 \times 2$ Factorial Design

Studure
Main Eliects
Simple Main Effects
Analysis a 2 2 Design number of scores that make up each cell.

- [ $T_{B_{1}}$ ] is the basic ratio of the total of the scores at level $B_{1}$ of factor $B$ : take the total of all the scores in level $B_{1}$ and square the total, divide the square by the number of scores making up this total.
- Eight basic ratios are required to test the four simple main effects ...


## Calculating Between-Group Sum of Squares

Sum of Squares between groups of factor $A$ at level $B_{1}\left(S S_{A \text { at } B_{1}}\right)$ :

$$
\left[A_{B_{1}}\right]-\left[T_{B_{1}}\right]
$$

Sum of Squares between groups of factor $A$ at level $B_{2}\left(S S_{A \text { at } B_{2}}\right)$ :

$$
\left[A_{B_{2}}\right]-\left[T_{B_{2}}\right]
$$

Sum of Squares between groups of factor $B$ at level $A_{1}\left(S S_{B \text { at } A_{1}}\right)$ :

$$
\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]
$$

$2 \times 2$ Factorial Design
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Main Elfectis
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Basic Ratios
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SS Main Effects SS interaction SSIn
DF
ANOVA Table
Simple Main Effects

Sum of Squares between groups of factor $B$ at level $A_{2}\left(S S_{B \text { at } A_{2}}\right)$ :

$$
\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]
$$

## Calculating Between－Group Degrees Of Freedom

－All degrees of freedom are equal to the number of（［number of levels in each simple main effect］）－ 1
－For the two simple main effects of $A$ ，the degrees of freedom are given by（ $a-1$ ），where $a$ is the number of levels in factor $A$
－For the two simple main effects of $B$ ，the degrees of freedom are given by（b－1），where $b$ is the number of levels in factor $B$

## Calculating Between－Group Sum of Squares

|  |  | Factor A：Fear |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Level $A_{1}$ no fear appeal | Level $A_{2}$ fear appeal |  |
| Efficacy | Level $B_{1}$ <br> no efficacy message | $\begin{gathered} \text { Total } A_{1} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } B_{1}= \\ 30+30=60 \end{gathered}$ |
|  | Level $B_{2}$ efficacy message | $\begin{gathered} \text { Total } A_{1} B_{2} \\ =31 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{2} \\ =49 \end{gathered}$ | $\begin{gathered} \text { Total } B_{2}= \\ 31+49=80 \end{gathered}$ |
|  |  | $\begin{gathered} \text { Total } A_{1}= \\ 30+31=61 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2}= \\ 30+49=79 \end{gathered}$ |  |

## Calculating Between－Group Sum of Squares

－Fear（no fear appeal vs．fear appeal）for no efficacy message（ $A$ at $B_{1}$ ）

$$
\left[A_{B_{1}}\right]=\frac{30^{2}+30^{2}}{6}=300\left[T_{B_{1}}\right]=\frac{60^{2}}{12}=300\left[A_{B_{1}}\right]-\left[T_{B_{1}}\right]=0
$$

－Fear（no fear appeal vs．fear appeal）for efficacy message（ $A$ at $B_{2}$ ）

$$
\left[A_{B_{2}}\right]=\frac{31^{2}+49^{2}}{6}=560.33\left[T_{B_{2}}\right]=\frac{80^{2}}{12}=533.33\left[A_{B_{2}}\right]-\left[T_{B_{2}}\right]=27
$$

Analysis a 2 2 Design
Data
Basic Ratios
SS WITHIN，BETWEEN，$\varepsilon$
TOTAL TOTAL
SS Main Effects
SS interaction
DF
ANOVA Table
Simple Main
Effects
Between－Group SS \＆DF

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$$

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$$

Simple Main Elfecis
Analysis a 2 2 Design
Data
Basic Ralios
SS WITHIN，BETWEEN，\＆ TOTAL
SS Main Eflecis
SS Interacion DF
ANOVA Table
Simple Main Effects
Between－Group SS \＆DF
Simple Main Effects Table

## Calculating Between－Group Sum of Squares

|  |  | Factor A：Fear |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Level $A_{1}$ no fear appeal | Level $A_{2}$ fear appeal |  |
| Efficacy | Level $B_{1}$ no efficacy message | $\begin{gathered} \text { Total } A_{1} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } B_{1}= \\ 30+30=60 \end{gathered}$ |
|  | Level $B_{2}$ efficacy message | $\begin{gathered} \text { Total } A_{1} B_{2} \\ =31 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{2} \\ =49 \end{gathered}$ | $\begin{gathered} \text { Total } B_{2}= \\ 31+49=80 \end{gathered}$ |
|  |  | $\begin{gathered} \text { Total } A_{1}= \\ 30+31=61 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2}= \\ 30+49=79 \end{gathered}$ |  |

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$$

Simple Main Elfecis
Analysis a 2 2 Design
Data
Basic Ralios
SS WITHIN，BETWEEN，\＆ TOTAL
SS Main Eflecis
SS Interacion DF
ANOVA Table
Simple Main Effects
Between－Group SS \＆DF
Simple Main Effects Table

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\left[A_{B_{2}}\right]=\frac{31^{2}+49^{2}}{6}=560.33\left[T_{B_{2}}\right]=\frac{80^{2}}{12}=533.33\left[A_{B_{2}}\right]-\left[T_{B_{2}}\right]=27
$$

Simple Main Efiecis
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SS WITHIN，BETWEEN，\＆ TOTAL
SS Main Eflecis
SS Interacion DF
ANoVA Table
Simple Main Effects
Between－Group SS \＆DF
Simple Main Effects Table

## Calculating Between-Group Sum of Squares

|  |  | Factor A: Fear |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Level $A_{1}$ no fear appeal | Level $A_{2}$ fear appeal |  |
| Efficacy | Level $B_{1}$ <br> no efficacy message | $\begin{gathered} \text { Total } A_{1} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } B_{1}= \\ 30+30=60 \end{gathered}$ |
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|  |  | $\begin{gathered} \text { Total } A_{1}= \\ 30+31=61 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2}= \\ 30+49=79 \end{gathered}$ |  |

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$$

－Fear（no fear appeal vs．fear appeal）for efficacy message（ $A$ at $B_{2}$ ）

$$
\left[A_{B_{2}}\right]=\frac{31^{2}+49^{2}}{6}=560.33\left[T_{B_{2}}\right]=\frac{80^{2}}{12}=533.33\left[A_{B_{2}}\right]-\left[T_{B_{2}}\right]=27
$$

Simple Main Efiecis
Analysis a 2 2 Design
Data
Basic Palios
SS WITHIN，BETWEEN，\＆ TOTAL
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Simple Main Effects
Between－Group SS \＆DF
Simple Main Effects Table

## Calculating Between－Group Sum of Squares

－Efficacy（no efficacy message vs．efficacy message）for no fear appeal（ $B$ at $A_{1}$ ）

$$
\left[B_{A_{1}}\right]=\frac{30^{2}+31^{2}}{6}=310.17\left[T_{A_{1}}\right]=\frac{61^{2}}{12}=310.08\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]=.09
$$

－Efficacy（no efficacy message vs．efficacy message）for fear appeal（ $B$ at $A_{2}$ ）

$$
\left[B_{A_{2}}\right]=\frac{30^{2}+49^{2}}{6}=550.17\left[T_{A_{2}}\right]=\frac{79^{2}}{12}=520.08\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]=30.09
$$

Analysis a 2 2 Design
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Basic Ratios
SS WITHIN，BETWEEN，$\varepsilon$
TOTAL TOTAL
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ANOVA Table
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Between－Group SS \＆DF Simple Main Effects Table

## Calculating Between-Group Sum of Squares

- Efficacy (no efficacy message vs. efficacy message) for no fear appeal ( $B$ at $A_{1}$ )

$$
\left[B_{A_{1}}\right]=\frac{30^{2}+31^{2}}{6}=310.17\left[T_{A_{1}}\right]=\frac{61^{2}}{12}=310.08\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]=.09
$$

- Efficacy (no efficacy message vs. efficacy message) for fear appeal ( $B$ at $A_{2}$ )

$$
\left[B_{A_{2}}\right]=\frac{30^{2}+49^{2}}{6}=550.17\left[T_{A_{2}}\right]=\frac{79^{2}}{12}=520.08\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]=30.09
$$

Simple Main Eliecis
Analysis a 2 2 Design Data
Basic Ratios
SS WITHIN, BETWEEN, $\varepsilon$ TOTAL
SS Main Effects
SS interaction DF
ANoVA Table
Simple Main Effects
Between-Group SS \& DF

## Calculating Between-Group Sum of Squares

|  |  | Factor A: Fear |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Level $A_{1}$ no fear appeal | Level $A_{2}$ fear appeal |  |
| Efficacy | Level $B_{1}$ no efficacy message | $\begin{gathered} \text { Total } A_{1} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } B_{1}= \\ 30+30=60 \end{gathered}$ |
|  | Level $B_{2}$ efficacy message | $\begin{gathered} \text { Total } A_{1} B_{2} \\ =31 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{2} \\ =49 \end{gathered}$ | $\begin{gathered} \text { Total } B_{2}= \\ 31+49=80 \end{gathered}$ |
|  |  | $\begin{gathered} \text { Total } A_{1}= \\ 30+31=61 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2}= \\ 30+49=79 \end{gathered}$ |  |

## Calculating Between-Group Sum of Squares

- Efficacy (no efficacy message vs. efficacy message) for no fear appeal ( $B$ at $A_{1}$ )

$$
\left[B_{A_{1}}\right]=\frac{30^{2}+31^{2}}{6}=310.17\left[T_{A_{1}}\right]=\frac{61^{2}}{12}=310.08\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]=.09
$$

- Efficacy (no efficacy message vs. efficacy message) for fear appeal ( $B$ at $A_{2}$ )

$$
\left[B_{A_{2}}\right]=\frac{30^{2}+49^{2}}{6}=550.17\left[T_{A_{2}}\right]=\frac{79^{2}}{12}=520.08\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]=30.09
$$

Simple Main Eliecis
Analysis a 2 2 Design Data
Basic Ratios
SS WITHIN, BETWEEN, $\varepsilon$ TOTAL
SS Main Effects
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Between-Group SS \& DF

## Calculating Between－Group Sum of Squares

－Efficacy（no efficacy message vs．efficacy message）for no fear appeal（ $B$ at $A_{1}$ ）

$$
\left[B_{A_{1}}\right]=\frac{30^{2}+31^{2}}{6}=310.17\left[T_{A_{1}}\right]=\frac{61^{2}}{12}=310.08\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]=.09
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$$
\left[B_{A_{2}}\right]=\frac{30^{2}+49^{2}}{6}=550.17\left[T_{A_{2}}\right]=\frac{79^{2}}{12}=520.08\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]=30.09
$$

Mimple Main Effects
Analysis a 2 2 Design Data
Basic Ratios
SS WITHIN，BETWEEN，\＆ TOTAL
SS Main Effects
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ANOVA Table
Simple Main Effects
Between－Group SS \＆DF

## Calculating Between-Group Sum of Squares

|  |  | Factor A: Fear |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Level $A_{1}$ no fear appeal | Level $A_{2}$ fear appeal |  |
| Factor $B$ <br> Efficacy | Level $B_{1}$ no efficacy message | $\begin{gathered} \text { Total } A_{1} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2} B_{1} \\ =30 \end{gathered}$ | $\begin{gathered} \text { Total } B_{1}= \\ 30+30=60 \end{gathered}$ |
|  | Level $B_{2}$ efficacy message | $\begin{aligned} & \text { Total } A_{1} B_{2} \\ & =31 \end{aligned}$ | $\begin{gathered} \text { Total } A_{2} B_{2} \\ =49 \end{gathered}$ | $\begin{gathered} \text { Total } B_{2}= \\ 31+49=80 \end{gathered}$ |
|  |  | $\begin{gathered} \text { Total } A_{1}= \\ 30+31=61 \end{gathered}$ | $\begin{gathered} \text { Total } A_{2}= \\ 30+49=79 \end{gathered}$ |  |

## Calculating Between－Group Sum of Squares

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$$
\left[B_{A_{1}}\right]=\frac{30^{2}+31^{2}}{6}=310.17\left[T_{A_{1}}\right]=\frac{61^{2}}{12}=310.08\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]=.09
$$

－Efficacy（no efficacy message vs．efficacy message）for fear appeal（ $B$ at $A_{2}$ ）

$$
\left[B_{A_{2}}\right]=\frac{30^{2}+49^{2}}{6}=550.17\left[T_{A_{2}}\right]=\frac{79^{2}}{12}=520.08\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]=30.09
$$

Mimple Main Effects
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SS Main Effects
SS interaction DF
ANOVA Table
Simple Main Effects
Between－Group SS \＆DF

## Summary Simple Main Effects Table By Components

| SOURCE | Sum of Squares | Degrees of freedom | Mean Square | F | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $A$ at $B_{1}$ | $\left[A_{B_{1}}\right]-\left[T_{B_{1}}\right]$ | $(a-1)$ | $\frac{\left[A_{B_{1}}\right]-\left[T_{B_{1}}\right]}{(a-1)}$ | $\frac{\text { Mean Square }_{A a t B_{1}}}{\text { Mean Square }_{S / A B}}$ | tables |
| $A$ at $B_{2}$ | $\left[A_{B_{2}}\right]-\left[T_{B_{2}}\right]$ | $(a-1)$ | $\frac{\left[A_{B_{2}}\right]-\left[T_{B_{2}}\right]}{(\mathrm{a}-1)}$ | $\frac{\text { Mean } \text { Square }_{A \text { at } B_{2}}}{\text { Mean Square }{ }_{S / A B}}$ | tables |
| $B$ at $A_{1}$ | $\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]$ | $(b-1)$ | $\frac{\left[B_{A_{1}}\right]-\left[T_{A_{1}}\right]}{(b-1)}$ | $\frac{\text { Mean Square }_{B_{\text {at } A_{1}}}}{\text { Mean Square }}$ | tables |
| $B$ at $A_{2}$ | $\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]$ | $(b-1)$ | $\frac{\left[B_{A_{2}}\right]-\left[T_{A_{2}}\right]}{(b-I)}$ | $\frac{\text { Mean Square }_{\mathrm{Bata}_{2}}}{\text { Mean Square }{ }_{S / A B}}$ | tables |
| $S / A B$ | $[Y]-[A B]$ | $a b(s-1)$ | $\frac{[Y]-[A B]}{a b(s-I)}$ |  |  | Design

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ss Man Ellects
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anova tebie
Simple Main Effects

Between-Group SS \& DF
Simple Main Effects Table

## Simple Main Effects Table For Hypothetical Data

| Source | Sum of Squares | Degrees of Freedom | Mean Square | $F$ | $P$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| $A$ at $B_{1}$ | 0.00 | 1 | 0.00 | 0.00 | 1.000 |
| $A$ at $B_{2}$ | 27.00 | 1 | 27.00 | 10.89 | $<.01$ |
| $B$ at $A_{1}$ | 0.09 | 1 | 0.09 | 0.04 | .856 |
| $B$ at $A_{2}$ | 30.09 | 1 | 30.09 | 12.13 | $<.01$ |
| $S / A B$ (error) | 49.67 | 20 | 2.48 |  |  |

ANOVA Table
Simple Main Effects
Between-Group SS \& DF
Simple Main Effects Table

## Interaction Plot


m.hurlstone@ lancaster.ac.uk
$2 \times 2$ Factorial Design
Siructure
Main Elects
Simple Main Effecis
Analysis a 2
2 Design
Dala
Basic Ralios
SS WITHIN, BETWEEN, \&
TOTAL
SS Main Effects
SS Interacion
DF
ANOVA Table
Simple Main
Effects
Between-Group SS \& DF
Simple Main Effects Table

## Efficacy

- No Efficacy Message ( $\mathrm{B}_{1}$ )
- Efficacy Message ( $\mathrm{B}_{2}$ )


## Additional Resources

－The $R$ code for all plots generated in this lecture（minus annotations） has been uploaded with these slides to the Week 6 lecture folder（ $R$ Plots For Lecture 7．R）

## In Next Week’s Lab ．．．

－Running a $2 \times 2$（and $2 \times 3$ ）between－participants ANOVA in R
－Calculating and interpreting simple main effects
Simple Main Effects
Analysis a 2

## References

Roberts, M. J., \& Russo, R. (1999, Chapter 9-10). A student's guide to Analysis of Variance. Routledge: London.

Main Effects
Simple Main Effects
Analysis a 2

