Factorial Validity

PSYC3302: Psychological Measurement and Its Applications

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Week 6

Learning Objectives

- Cronbach's α assumption revisited
- Introduce factor analysis
    - Unidimensional
    - Multidimensional (correlated)
    - Multidimensional (uncorrelated)
- Conducting and Interpreting a PCA
    - Scree plot
    - Communalities
    - Eigenvalues
    - Component Loadings
    - Simple Structure
    - Component correlations

Previously ...

- Internal consistency reliability is typically estimated with Cronbach's α
- Cronbach's α assumes unidimensionality
  - composite scores represent one dimension or one construct
- How does one determine whether a group of items actually measure a single dimension?
- We use a technique known broadly as "factor analysis"

Example

- Consider the items below which form a fictitious, 6-item test:
  1. I enjoy socialising with a large number of people
  2. I spend time pondering philosophical questions
  3. I'm happy to lead a discussion
  4. I like to entertain novel solutions to intellectual problems
  5. I have a large social group
  6. Spending a Friday night with a good book is fine by me

- Can you detect a pattern?
  - Items 1, 3, and 5 appear to measure a socialisation dimension
  - Items 2, 4, and 6 appear to measure an intellectual dimension

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Factorial Validity and Factor Analysis

- Factorial Validity is relevant to determining whether the <u>number and nature of dimensions</u> associated with scores from a test corresponds to the theorised number and nature of dimensions underlying the construct of interest
- Researchers use a data analytic technique known as factor analysis to help determine the number and nature of dimensions associated with the scores derived from a test

Factorial Validity

- Factorial validity concerns the internal structure of test scores
- A test's internal structure is the way that the parts of the test are related to each other
- The actual (empirical) structure of the test should match the theoretical or intended structure of the test
- In a very simplistic sense, factorial validity helps specify what the test measures
- That is, with respect to the number of dimensions, and the definition of those dimensions
- The definitions of the dimensions are determined by which items "load" onto which dimension

Factorial Validity

- To determine the number of dimensions that underlie a test, as well as their definitions, we (can) use a data analytic technique known as factor analysis
- Factor analysis ...
  - helps us clarify the number of factors within a set of items
  - helps us determine the nature of the associations among the factors
  - helps us determine which items are linked to which factor, which facilitates the interpretation of those factors

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Measurement

- Typically, when we attempt to measure an attribute, we try to do so in such a way that the score represents a single attribute
- Consider the example earlier based on the six-items. If I calculated a total score based on those six items, what would that composite score mean?
- It would be a compromised score, because the items do not all measure the same dimension
- In practice, we try to avoid such situations
- We want composite scores that are relatively "factor pure"

Test Dimensionality

- As a general statement, there are three different types of tests:
  1. Unidimensional test
     - consists of items which all measure one, single factor
  2. Multidimensional test (correlated)
     - consists of items which measure two or more dimensions which are correlated with each other (positively or negatively)
  3. Multidimensional test (uncorrelated)
     - consists of items which measure two or more dimensions which are unrelated to each other

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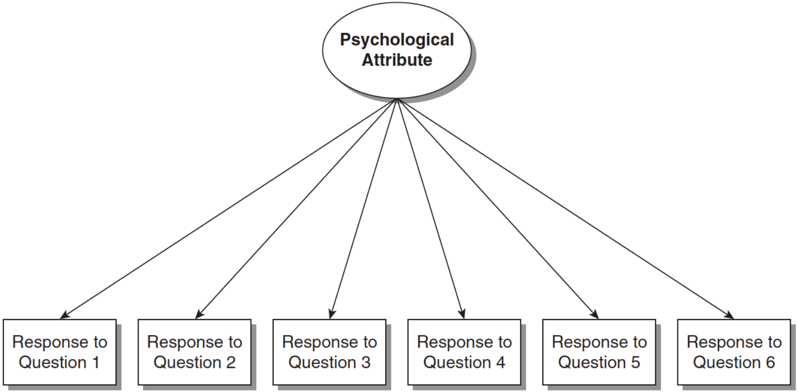
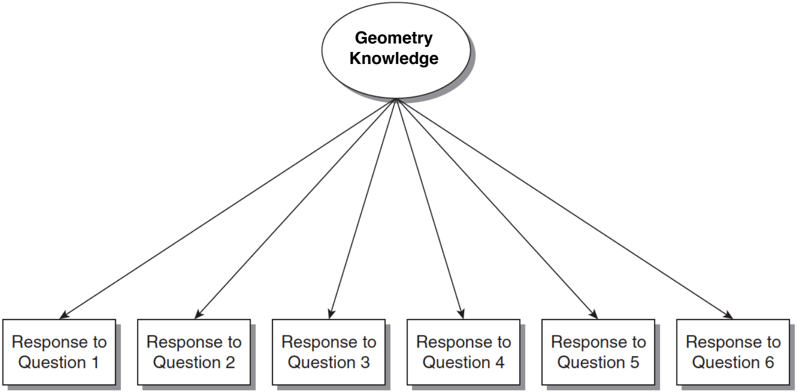
Unidimensional Tests

- A test is unidimensional when it includes items that reflect a single psychological attribute
- This means that responses to those items are driven only by that attribute
- For example, a student's score on a multiple-choice geometry exam is interpreted as a measure of the amount of his or her "knowledge of geometry"
- Provided that the test only requires knowledge of geometry—and not algebra or calculus—such a test would be unidimensional
- The test items would also have the property of *conceptual homogeneity*

Unidimensional Tests



Unidimensional Tests



Unidimensional Tests

- A test's dimensionality has implications for its scoring, evaluation, and use
- For a unidimensional test, only a single score is computed, reflecting the single psychological attribute measured by the test
- That is, all the items are combined in some way (usually through averaging, summing, or counting) to form a composite or "total" score
- For unidimensional tests, reliability and validity should be estimated and evaluated for the total score produced by the test

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Multidimensional Tests With Correlated Dimensions: Tests With Higher Order Factors

- When a psychological test includes items reflecting more than one psychological attribute, the test is considered multidimensional
- If a test has multiple dimensions that are correlated with each other, the test can be considered a multidimensional test with correlated dimensions
- the Stanford-Binet (SB5) intelligence test is a multidimensional test with correlated dimensions
- It includes groups of questions (known as subtests) that assess different psychological attributes: (1) fluid reasoning, (2) general knowledge, (3) quantitative processing, (4) visual-spatial processing, and (5) working memory

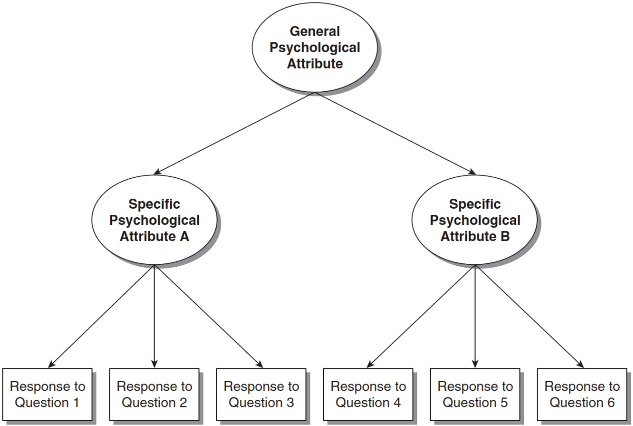
Multidimensional Tests With Correlated Dimensions: Tests With Higher Order Factors

- Multidimensional tests with correlated dimensions can produce a variety of scores
- Typically, each subtest has its own subtest score
- Each subtest is, itself, unidimensional, and the questions in each subtest are conceptually homogeneous
- For example, the quantitative processing subtest of the SB5 might require a test taker to answer 10 questions
- Responses to each of those 10 questions reflect only quantitative processing and not one of the constructs represented by the other subtests

Multidimensional Tests With Correlated Dimensions: Tests With Higher Order Factors

- In addition to scores for each subtest, multidimensional tests with correlated dimensions are often scored in a way that produces a total score, combined across several subtests
- That is, subtest scores are often combined with each other (again, either through summing or by averaging the scores) to produce a total test score
- For example, the five subtest scores from the SB5 are combined to form an overall "full-scale" score representing general intelligence, or g (a general psychological attribute)

Multidimensional Tests With Correlated Dimensions: Tests With Higher Order Factors



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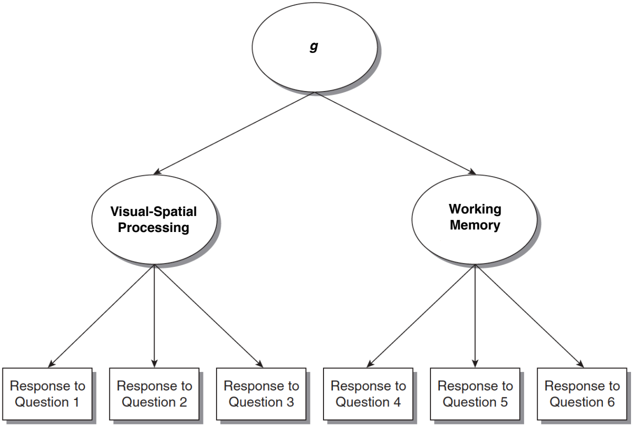

Multidimensional Tests With Correlated Dimensions: Tests With Higher Order Factors

- In terms of test evaluation, multidimensional tests have a score for each subtest, and each subtest score is evaluated with regard to its psychometric quality
- For example, the developers and users of the SB5 have examined carefully the reliability and validity of each of its five subtests
- Additionally, a multidimensional test with correlated dimensions may have a total test score that is computed across its subtests, and this total score also requires psychometric evaluation
- For example, the developers and users of the SB5 have examined the reliability and validity of its full-scale score

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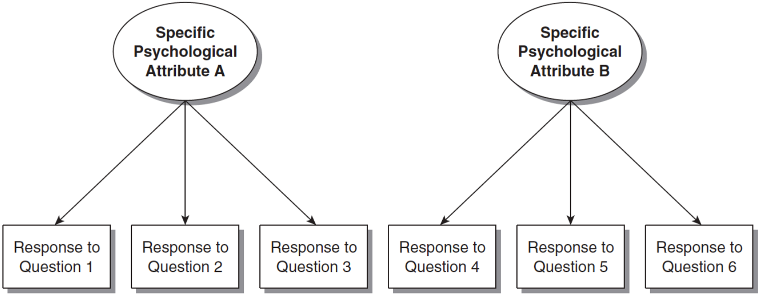
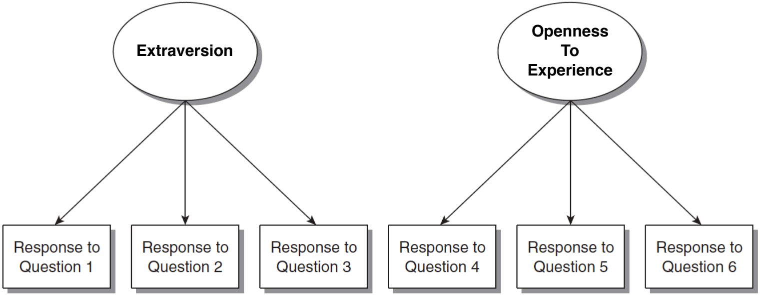
Multidimensional Tests With Uncorrelated Dimensions

- If a test's dimensions are not associated with each other (or are only weakly associated with each other), then the test can be considered a multidimensional test with uncorrelated dimensions
- Several personality tests are multidimensional with dimensions that are generally treated as if they are uncorrelated
- For example, a test called the NEO Five Factor Inventory (NEO-FFI; Costa McCrae, 1992) is a 60-item questionnaire reflecting five dimensions, or factors of personality
- The NEO-FFI is designed to measure five relatively independent personality attributes, and these five attributes are not typically treated as reflecting any higher-order factors

Multidimensional Tests With Uncorrelated Dimensions



Multidimensional Tests With Uncorrelated Dimensions



Multidimensional Tests With Uncorrelated Dimensions

- With regard to scoring, evaluation, and use, multidimensional tests with uncorrelated dimensions are similar to multidimensional tests with correlated dimensions
- There is one important exception—no total test score is computed
- That is, a score is obtained for each dimension, but the dimensions scores are not combined to compute a total test score
- Furthermore, each of the dimension scores is evaluated in terms of psychometric quality
- For example, the NEO-FFI produces only five scores—one for each of the five factors or dimensions—no total test score is computed

Factor Analysis: Examining The Dimensionality of a Test

- Test developers can use a variety of statistical procedures to evaluate a test's dimensionality
- Although procedures such as cluster analysis and multidimensional scaling are available, **factor analysis** is the most common method of examination
- There are, in fact, two broad types of factor analysis: exploratory factor analysis (EFA) and confirmatory factor analysis (CFA)
- EFA is the more common type, and is the one we will focus on here

Factor Analysis: Examining The Dimensionality of a Test

- In the textbook, they mention six personality attributes:
  - Talkative
  - Assertive
  - Outgoing
  - Creative
  - Imaginative
  - Intellectual
- How many dimensions do these 6 attributes measure?
- You could have qualitative discussions with people about this, but that is not a very scientific approach to answering the question

Factor Analysis: Examining The Dimensionality of a Test

- Instead, you could get about 100 people to respond to each adjective on a 5-point scale from "Very much not like me" to "Very much like me"
- Then, you could at the very least examine the pattern of correlations between the items

	Talkative	*Assertive*	*Outgoing*	*Creative*	*Imaginative*	*Intellectual*
Talkative	1.00					
Assertive	.66	1.00				
Outgoing	.54	.59	1.00			
Creative	.00	.00	.00	1.00		
Imaginative	.00	.00	.00	.46	1.00	
Intellectual	.00	.00	.00	.57	.72	1.00

Factor Analysis: Examining The Dimensionality of a Test

- It can be observed that the first three adjectives cluster together and the last three adjectives cluster together
- There are no correlations between the first cluster of adjectives and the second cluster of adjectives
- Thus, it appears that this six item test is measuring two uncorrelated dimensions

	Talkative	*Assertive*	*Outgoing*	*Creative*	*Imaginative*	*Intellectual*
Talkative	1.00					
Assertive	.66	1.00				
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Creative	.00	.00	.00	1.00		
Imaginative	.00	.00	.00	.46	1.00	
Intellectual	.00	.00	.00	.57	.72	1.00

Factor Analysis: Examining The Dimensionality of a Test

- This is an eye-ball approach to conducting a factor analysis
- In practice, it is not especially useful, because:
  - there are usually a lot more than 6 variables included in a factor analysis
  - the pattern of correlations is not usually as clear cut as those in the above table
- It is more sophisticated and efficient to conduct an actual factor analysis

Factor Analysis: Examining The Dimensionality of a Test

- The textbook makes clear that what they conducted was technically a **principle component analysis** rather than a factor analysis
- There are differences between component analysis and factor analysis, but for the purposes of this unit, you do not have to know them
- In practice, there are usually not very big differences between the results of a principal component analysis (PCA) and a factor analysis
- The textbook reports the results of the PCA based on a software package known as SAS
- But let's go through them in SPSS

Conducting a PCA

- In practice, you would conduct the PCA twice:
  - once to determine the number of components to extract
  - then again to determine the number of components that you want to extract
- The textbook does not make this clear

SPSS Analysis

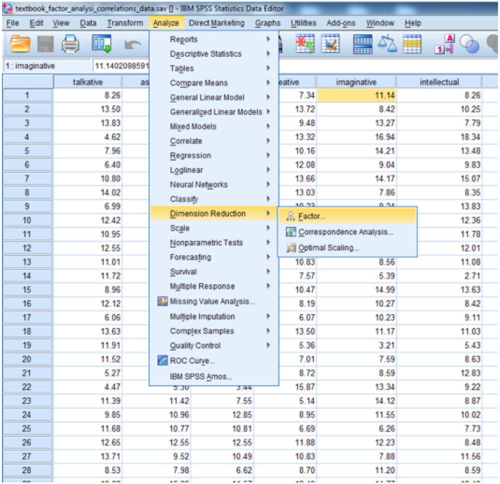
textbook_factor_analysi_correlations_data.sav [] - IBM SPSS Statistics Data Editor

File Edit View Data Transform Analyze Direct Marketing Graphs Utilities Add-ons Window Help

1 : talkative 8.25727586374112

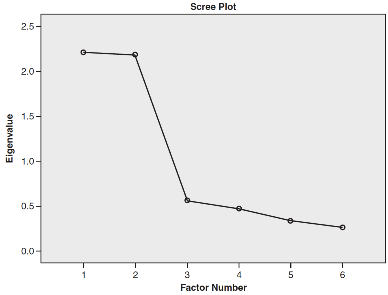
	talkative	assertive	outgoing	creative	imaginative	intellectual
1	8.26	10.40	13.23	7.34	11.14	8.26
2	13.50	9.61	10.01	13.72	8.42	10.25
3	13.83	12.72	12.12	9.48	13.27	7.79
4	4.62	6.45	6.60	13.32	16.94	18.34
5	7.96	1.82	7.21	10.16	14.21	13.48
6	6.40	7.09	3.17	12.08	9.04	9.83
7	10.80	11.49	8.46	13.66	14.17	15.07
8	14.02	11.17	13.98	13.03	7.86	8.35
9	6.99	9.72	7.46	10.23	9.24	13.83
10	12.42	8.05	13.17	13.59	12.31	12.36
11	10.95	13.76	8.17	10.30	9.51	11.78
12	12.55	15.17	16.86	7.56	11.79	12.01
13	11.01	6.95	9.02	10.83	8.56	11.08
14	11.72	14.12	12.46	7.57	5.39	2.71
15	8.96	8.98	10.24	10.47	14.99	13.63
16	12.12	8.55	9.35	8.19	10.27	8.42
17	6.06	7.13	5.88	6.07	10.23	9.11
18	13.63	13.15	10.60	13.50	11.17	11.03
19	11.91	10.30	9.03	5.36	3.21	5.43
20	11.52	10.40	10.16	7.01	7.59	8.63
21	5.27	5.08	8.33	8.72	8.59	12.83
22	4.47	5.30	3.44	15.87	13.34	9.22
23	11.39	11.42	7.55	5.14	14.12	8.87
24	9.85	10.96	12.85	8.95	11.55	10.02
25	11.68	10.77	10.81	6.69	6.26	7.73
26	12.65	12.55	12.55	11.88	12.23	8.48
27	13.71	9.52	10.49	10.83	7.88	11.56
28	8.53	7.98	6.62	8.70	11.20	8.59
29	10.62	16.26	11.57	12.49	14.77	12.40

SPSS Analysis



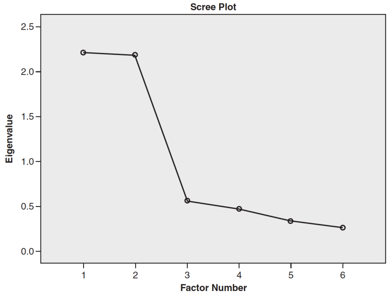
Preliminary Analysis: Scree Plot

- SPSS produces a graph known as a "scree plot"
- A scree plot consists of eigenvalues ordered from smallest to largest in a scatter plot
- Eigenvalues are essentially numerical representation of components with respect to their size
- The larger an eigenvalue, the more likely it is representing a worthwhile component



Preliminary Analysis: Scree Plot

- When examining a scree plot, you are hoping to find a clear break in the plot
- In this case, it can be observed that there is a clear break after the first two eigenvalues
- Eigenvalues 3 to 6 would be considered part of the "scree"
- They are basically error variance, and, therefore not worthwhile components to investigate (or extract) for the purposes of interpretation



Factor Extraction

- Now that I know how many components to extract from the analysis, I will redo the analysis in SPSS with the specification of 2 components

Factor Extraction: Communalities

- The communality for a given variable can be interpreted as the **percentage of variation** in that variable explained by the extracted components
- In this case, it can be seen that 73.9% of the talkative variable's variance has been accounted for by the PCA
- As a general statement, you want to see communalities that are **at least .04 or .09**
- Depending on the type of data you are analysing:
    - .04 or greater for PCA based on items
    - .09 or greater for PCA based on sub-scales
- Items are less reliable than subscales, so they have lower communality expectations

Communalities

	Initial	Extraction
talkative	1.000	.739
assertive	1.000	.777
outgoing	1.000	.679
creative	1.000	.607
imaginative	1.000	.743
intellectual	1.000	.823

Extraction Method: Principal Component Analysis.

Factor Extraction: Eigenvalues

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings[a]
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total
1	2.195	36.578	36.578	2.195	36.578	36.578	2.195
2	2.173	36.222	72.800	2.173	36.222	72.800	2.173
3	.563	9.382	82.183				
4	.472	7.867	90.050				
5	.333	5.554	95.604				
6	.264	4.396	100.000				

Extraction Method: Principal Component Analysis.

a. When components are correlated, sums of squared loadings cannot be added to obtain a total variance.

- SPSS gives the eigenvalues and the percentage of variance associated with each
- It separates the results into two halves: Unrotated ("Initial") and Rotated ("Rotation Sums of Squared Loadings")
- The sum of the "Initial Eigenvalues" sums to 6, because this is the number of variables included in the analysis

Factor Extraction: Eigenvalues

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings[a]
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Extraction Method: Principal Component Analysis.

a. When components are correlated, sums of squared loadings cannot be added to obtain a total variance.

- It can be seen that the first component has accounted for 36.6% of the total variance in the analysis
- The second component has accounted for 36.2% of the total variance in the analysis
- On a cumulative basis therefore, the two components have accounted for 72.8% of the total variance

Factor Extraction: Component Loadings

- Component loadings refer to the associations between items and components and they are the key to understanding the psychological meaning of a component
- These loadings vary between –1 and +1
- The size of the loading indicates the degree of association between an item and a component—larger loadings (values further from 0) indicate stronger associations
- A positive loading indicates that people who respond with a high score on an item have a high level of the underlying component
- A negative loading indicates that people who respond with a high score on an item have a low level of the underlying component

Factor Extraction: Component Loadings

- SPSS has extracted two components, therefore, there are two columns of component loadings
- As a general statement, useful component loadings are either .20 or .30 or greater
  - .20 for PCA based on items
  - .30 for PCA based on sub-scales

Pattern Matrix[a]

	Component 1	Component 2
talkative	.859	.000
assertive	.882	.000
outgoing	.824	.000
creative	.000	.779
imaginative	.000	.862
intellectual	.000	.907

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 1 iterations.

Factor Extraction: Component Loadings

- The first component consists of loadings from the *assertive*, *talkative*, and *outgoing* variables
- It could be called an **Extraversion** dimension (or component)
- By contrast, the second component is defined by the *intellectual*, *imaginitive*, and *creative* variables
- We might label this dimension (or component) **Openness to experience**

Pattern Matrix[a]

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Factor Extraction: Simple Structure

- These component loadings are an ideal example of "simple structure"
- Simple structure occurs when each item is **strongly linked to one and only one component**
- It can be seen that each item loads robustly on one component but has a loading of .000 on the other component
- Thus, each item clearly belongs on one and only one component
- *This level of simple structure is not be expected in practice—it results from the fact that the correlations presented at the outset were created to be as clear as possible*

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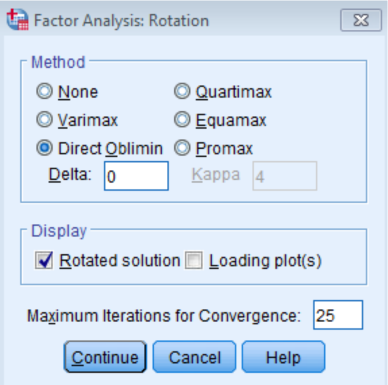
Factor Extraction: Simple Structure

- One way to help achieve simple structure is to rotate the solution
- You should always rotate the solution
- In this unit, always use **Direct Oblimin**
- You can only rotate a solution if you extract two or more components



Factor Extraction: Component/Pattern/Structure Matrices

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Extraction Method: Principal Component Analysis.

a. 2 components extracted.

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Structure Matrix

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intellectual	.000	.907

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

- If you extract more than one component and you rotate the solution, SPSS will give you three tables with component loadings
- In this unit, only ever focus on the "Pattern Matrix"
- If you extract only one component, then there will only be the "Component Matrix"
- *Notice that the component loadings are identical in the three matrices—this is an atypical result which is again due to the way the data were constructed*

Component Correlations

Component Correlation Matrix

Component	1	2
1	1.000	.000
2	.000	1.000

Extraction Method: Principal Component Analysis.
Rotation Method: Oblimin with Kaiser Normalization.

- It can be observed that the correlation between the two components is equal to 0
- They are two totally separate dimensions with no overlap
- You cannot predict how high someone might score on "openness to experience" if you know how high they were on "extraversion"

Sample Size Requirements

- There are no simple guidelines to follow, because the size of the sample required will be dependent upon two main factors:
  - the amount of communality associated with the variables (higher communality means less sample size required)
  - the number of variables per factor (higher number of variables per factor means less sample size required)
- If you have a minimum of 5 variables per factor and the communalities all exceed .20, then a sample size of about 150 should be sufficient

Factor Analysis

- You will learn more about the execution of a principal component analysis in the lab next week

References

Furr, M. R., & Bacharach, V. R. (2014; Chapter 9). Psychometrics: An Introduction (second edition). Sage.