Behavioural Economics

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Outline

The Standard Model

Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

Behavioural Economics

PSYC3310: Specialist Topics In Psychology

Mark Hurlstone Univeristy of Western Australia

Seminar 4: Judgement Under Risk & Uncertainty



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Last week

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Outline

- The Standard Model Perfect Rationality
- Bayesian Probabi Estimation

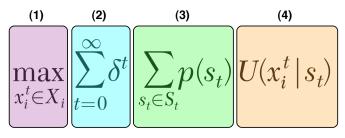
Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoker Word Recognition

Next ...

• Last week, we focused on preferences and choices—components **1** and **4** in the standard model



Today

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Outline

The Standard Model Perfect Rationality Bayesian Probability Estimation

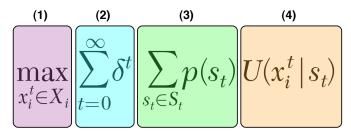
Bayes' Rule

Example 1: Media Diagnosis

Example 2: Spoker Word Recognition

Next ...

• Examine probabilities and beliefs in the standard model—component 3



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- Judgement under risk & uncertainty
 - confront the model with empirical data

Today

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Outline

The Standard Model Perfect Rationality Bayesian Probability Estimation

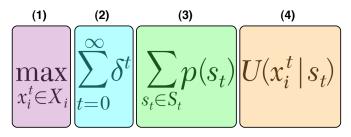
Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoker Word Recognition

Next ...

• Examine probabilities and beliefs in the standard model—component 3



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3

- Judgement under risk & uncertainty
 - confront the model with empirical data

The standard model: An example

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Outline

The Standard Model

Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

Next ...

Suppose you have to choose between two 3310 topics:

- (1) Behavioural Economics (BE)
- (2) Cognition and Emotion (CE)

Choice	State of world $s \in S$	Probability $p(s)$	Utility $U(x s)$
BE	Exciting	0.8	60
	Dull	0.2	30
CE	Exciting	0.05	60
	Dull	0.95	30

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 $U(BE) = (0.8 \times 60) + (0.2 \times 30) = 54$ $U(CE) = (0.05 \times 60) + (0.95 \times 30) = 31.5$

The standard model: An example

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Bayes' Rule

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Choice	State of world $s \in S$	Probability $p(s)$	Utility $U(x s)$
BE	Exciting	0.3	60
	Dull	0.7	30
CE	Exciting	0.75	60
	Dull	0.25	30

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 $U(\mathsf{BE}) = (0.3 \times 60) + (0.7 \times 30) =$ **39** $U(\mathsf{CE}) = (0.75 \times 60) + (0.25 \times 30) =$ **52.5**

The standard model: An example

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The standard model: Main assumptions

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Outline

The Standard Model

Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

- In terms of beliefs, the standard model makes two central assumptions
 - Perfect rationality
 - Bayesian probability estimation
- Let's look at each in turn ...

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The standard model: Perfect rationality

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Outline

- The Standarc Model
- Perfect Rationality
- Bayesian Probabili Estimation

Bayes' Rule

- Example 1: Medica Diagnosis
- Example 2: Spoken Word Recognition

- We've already discussed this in the past two seminars
- The basic assumptions here are that
 - people have all the relevant information they need when they make a decision
 - they have the cognitive resources to process it instantly and without cost
- **Bounded rationality** (Simon, 1955) is the term we use if these assumptions do not hold

The standard model: Bayesian probability estimation

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- Outline
- The Standar Model
- Perfect Rationality
- Bayesian Probability Estimation
- Bayes' Rule
- Example 1: Medic Diagnosis
- Example 2: Spoken Word Recognition
- Next ...

- People are assumed to be Bayesian probability estimators
- This means that
 - they are able to estimate probabilities correctly, given the relevant information
 - they are able to update these probabilities in light of new information
- In order to understand this assumption, we need to introduce **Bayes' rule**

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The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

- Bayes' rule is a rigorous method for interpreting evidence in the context of previous experience or knowledge
- It was discovered by the English statistician and minister, Thomas Bayes (1701-1761)



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The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

Next ...

- Bayes' rule, or *Bayes' theorem*, constitutes a mathematical foundation for reasoning
- It is not a matter of conjecture—as a theorem it has been proved to be true
- In essence, Bayes' rule is used to combine prior experience with observed data to interpret these data

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This process is known as Bayesian inference

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The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

Next ...

• Bayes' rule can be specified as follows

$$p(hypothesis|data) = \frac{p(data|hypothesis) \times p(hypothesis)}{p(data)}, \quad (1)$$

Where:

- *p*(*hypothesis*|*data*) = *posterior probability*
- p(data|hypothesis) = likelihood
- p(hypothesis) = prior probability
- p(data) = marginal likelihood
 - calculated as $\sum p(data|hypothesis_i) \times p(hypothesis_i)$

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The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

Next ...

Posterior probability

• the probability the hypothesis is true given the data

Likelihood

 the likelihood of observing the data if the hypothesis is true

Prior probability

the prior probability of the hypothesis

Marginal likelihood

• the *a priori* probability of witnessing the data under all hypotheses

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Outline

The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

- You wake up one day with spots on your face
- Your doctor informs you the symptoms (*data*) are consistent with two Poxy diseases (*hypotheses*): smallpox and chickenpox
- She knows 80% of people with chickenpox have spots, but also that 90% of people with smallpox have spots
- How should she decide between the two diagnoses?

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Outline

The Standard Model

Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

- Although the probability of spots given you have chickenpox or smallpox is comparable, your doctor knows the former disease is common, but the latter is rare
- This *prior information* can be used to determine which disease you might have
- Reaching a decision requires the doctor to combine the possible diagnoses (hypotheses) implied by your symptoms (data) with her prior knowledge (prior information)

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Outline

The Standard Model Perfect Rationality

Devere? Device

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

 Based on medical reports, your doctor knows the following

- The conditional probability a patient has spots given they have smallpox is 90% or 0.9
 - p(spots|smallpox) = 0.9,
- The conditional probability a patient has spots given they have chickenpox is 80% or 0.8
 - p(spots|chickenpox) = 0.8,
- So far this does not take into account prior information about the relative prevalence of the two diseases

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The Standard Model

Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

- Public health statistics indicate the prevalence of smallpox in the general population is 0.001
 - p(smallpox) = 0.001,
- The prevalence of chickenpox in the general population is 0.1
 - p(chickenpox) = 0.1,
- The probability that a randomly chosen person in the population has spots—the marginal likelihood—is (0.9 × 0.001) + (0.8 × 0.1) = 0.081
 - p(spots) = 0.081,
- Your doctor now has the prior information she needs to make a diagnosis using Bayes' rule

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Outline

The Standard Model Perfect Rationality Bayesian Probability

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

Using Bayes' rule, the *posterior probability* that you have smallpox, given your symptoms, can be calculated as:

$$p(smallpox|spots) = \frac{p(spots|smallpox) \times p(smallpox)}{p(spots)}, \quad (2)$$

Plugging in the values from the previous slides we get

$$p(smallpox|spots) = 0.9 \times 0.001/0.081$$
 (3)

$$p(smallpox|spots) = 0.011.$$
 (4)

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The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

Now let's calculate the *posterior probability* that you have chickenpox:

$$p(chickenpox|spots) = \frac{p(spots|chickenpox) \times p(chickenpox)}{p(spots)},$$
(5)

Plugging in the values from the previous slides we get

$$p(chickenpox|spots) = 0.8 \times 0.1/0.081$$
(6)

p(chickenpox|spots) = 0.988. (7)

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Outline

The Standard Model Perfect Rationality Bayesian Probabilit Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

The two posterior probabilities are:

$$p(smallpox|spots) = 0.011.$$
(8)

$$p(chickenpox|spots) = 0.988.$$
 (9)

Using these values we can create a posterior ratio

$$R_{post} = \frac{p(chickenpox|spots)}{p(smallpox|spots)}$$
(10)

$$R_{post} = \frac{0.988}{0.011} \tag{11}$$

$$R_{post} = 89.81$$
 (12)

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Outline

The Standard Model

Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

Next ...

Let's consider another example, but before that some light hearted relief Two Ronnies Sketch

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Outline

The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

- Suppose you are a shopkeeper and a customer asks you *have you got forkandles?*
- The acoustic input is ambiguous: did the customer ask for *four candles* or *fork handles*?
- To adjudicate between the two hypotheses, you may use prior information to infer the correct interpretation
- You sell many more candles than fork handles
- Thus, you may not hear the words *fork handles* and instead hear *four candles*

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Outline

The Standard Model

Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

- Lets make the following assumptions for our example
- The probability that the phrase spoken was *four candles* is 0.6
 - p(data|four candles) = 0.6,
- The probability that the phrase spoken was *fork handles* is 0.7
 - p(data|fork handles) = 0.7,

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Outline

- The Standard Model Perfect Rationality
- Bayesian Probabili Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

- You have been asked 90 times in the past for four candles and only 10 times for fork handles
- Before the customer speaks, you estimate that the probability he will say each phrase is
 - p(four candles) = 0.9,
 - p(fork handles) = 0.1,
- The marginal likelihood is (0.6 \times 0.9) + (0.7 \times 0.1) = 0.61
 - p(data) = 0.61

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Outline

The Standard Model Perfect Rationality Bayesian Probability Estimation

Bayes' Rule

Example 1: Medic Diagnosis

Example 2: Spoken Word Recognition

Next ...

Using Bayes' rule, the posterior probability that the customer said *four candles* is: -

$$p(four \ candles|data) = \frac{p(data|four \ candles) \times p(four \ candles)}{p(data)},$$
(13)

Plugging in the values from the previous slides we get

$$p(four \, candles | data) = 0.6 \times 0.9 / 0.61 \tag{14}$$

 $p(four \ candles | data) = 0.885$ (15)

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Outline

The Standard Model Perfect Rationality Bayesian Probability Estimation

Example 1: Media

Example 2: Spoker

Next ...

The posterior probability that the customer said *fork handles* is: -

$$p(fork handles|data) = \frac{p(data|fork handles) \times p(fork handles)}{p(data)},$$
(16)

Plugging in the values from the previous slides we get

$$p(fork handles|data) = 0.7 \times 0.1/0.61$$
(17)

p(fork handles|data) = 0.115 (18)

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Outline

The Standard Model Perfect Rationality Bayesian Probabili Estimation

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

Next ...

The two posterior probabilities are:

$$p(four \, candles | data) = 0.885$$
 (19)

$$p(fork handles|data) = 0.115$$
 (20)

Using these values we can create a posterior ratio

$$R_{post} = \frac{p(four \, candles | data)}{p(fork \, handles | data)}$$
(21)

$$R_{post} = \frac{0.885}{0.115} \tag{22}$$

$$R_{post} = 7.69 \tag{23}$$

Next... are people Bayesian probability estimators?

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Outline

The Standard Model Perfect Rationality Bayesian Probability

Bayes' Rule

Example 1: Medica Diagnosis

Example 2: Spoken Word Recognition

- Do people behave according to the rules of logic and probability theory as the standard model predicts?
- Do they update their beliefs using Bayes' rule (so-called **Bayesian updating**)?
- · We consider instances where this is not the case
 - speaker 1: gambler's and conjunction fallacies
 - speaker 2: base-rate neglect and planning fallacy
 - speaker 3: confirmation bias
 - speaker 4: availability heuristic and hindsight bias