Factorial Designs

PSYC204: Experimental Research Methods

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Week 4

Learning Objectives

- Introduction to factorial designs
- Main effects and interactions
  - Identifying interactions
  - Interpreting main effects and interactions
  - Independence of main effects and interactions
- Types of factorial designs
  - Between- and within-participants designs
  - Mixed designs
  - Pre- and post-test designs
  - Higher-order factorial designs
- Statistical analysis of factorial designs

Factorial Designs

Factorial Designs

- So far, we have focused on situations involving one independent variable and one dependent variable
- However, behaviour is usually influenced by a variety of different variables acting and interacting simultaneously
- To examine these more complex situations, we often design studies that include more than one independent variable

Today:

- Introduction to **factorial designs**
- Designs incorporating two or more independent variables

Example: Ackerman & Goldsmith (2011)

- Compared the effectiveness of presentation format and study time on retention of information presented in text
- Two independent variables:
  1. presentation format
     - on paper vs. on screen
  2. study time
     - fixed vs. self regulated
- One dependent variable:
  - exam scores a subsequent multiple choice test of the studied material

Matrix Representation of Conditions

Presentation Format

Study Time	On Paper	On Screen
Fixed	Exam scores for a group of participants who studied text presented on paper for a fixed time.	Exam scores for a group of participants who studied text presented on screen for a fixed time.
Self Regulated	Exam scores for a group of participants who studied text presented on paper for a self-regulated time.	Exam scores for a group of participants who studied text presented on screen for a self-regulated time.

Terminology of Factorial Designs

- When two or more independent variables are combined in a study, the independent variables are called **factors**:
  - In the study of Ackerman and Goldsmith (2011), there are two factors; presentation format and study time
- A study involving two or more factors is called a **factorial design**
- This type of design is referenced by the number of its factors (e.g., two-factor design, three-factor design etc.):
  - The Ackerman and Goldsmith (2011) study is a **two-factor design**
  - A study with a single independent variable is called a **single-factor design**

Terminology of Factorial Designs

- Factors are referenced by their name (e.g., presentation format, study time)
- A notation system is used to indicate the number of **levels** of each factor:
  - Our example study has two levels for the presentation-format factor, and two levels for the study-time factor
  - It can be described as a 2×2 (read as "two by two") factorial design
- The total number of treatment conditions can be determined by multiplying the levels for each factor:
  - A $2 \times 3 \times 2$ design is a **three-factor design** with two, three, and two levels of each of the three factors (12 conditions in total)

Main Effects and Interactions

Main Effects and Interactions

- A factorial design allows researchers to examine how unique combinations of factors acting together influence behaviour
- We illustrate this using the simplest possible factorial design, the two-factor design
- The data from a factorial study generates two sources of information:
    1. Main effects
    2. Interaction between factors

Main Effects

- The mean differences among the levels of one factor are called the **main effect** of that factor
- Main effects provide information about the independent effects of each factor
- A two factor study has two main effects, one for each of the two factors
- When a study is represented as a matrix:
  - mean differences among the *columns* define the main effect one factor
  - mean differences among the *rows* define the main effect for the second factor

Main Effects

- We consider some hypothetical data for the paper/on-screen study to illustrate main effects

Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 22	M = 18	Overall M = 20
Self Regulated	M = 18	M = 14	Overall M = 16
	Overall M = 20	Overall M = 16	

Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 22	M = 18	Overall M = 20
Self Regulated	M = 18	M = 14	Overall M = 16

Overall M = 20 ⟷ Overall M = 16

Main effect of presentation format:
20-16 = 4

Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	*M* = 22	*M* = 18	Overall *M* = 20
Self Regulated	*M* = 18	*M* = 14	Overall *M* = 16

Overall *M* = 20 ⟷ Overall *M* = 16

Main effect of presentation format:
20-16 = 4

Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 22	M = 18	Overall M = 20
Self Regulated	M = 18	M = 14	Overall M = 16
	Overall M = 20	Overall M = 16	

Overall M = 20 ⟷ Overall M = 16

Main effect of presentation format:
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Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	$M = 22$	$M = 18$	Overall $M = 20$
Self Regulated	$M = 18$	$M = 14$	Overall $M = 16$
	Overall $M = 20$	Overall $M = 16$	

Main effect of study time:
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Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 22	M = 18	Overall M = 20
Self Regulated	M = 18	M = 14	Overall M = 16
	Overall M = 20	Overall M = 16	

Main effect of study time:
20-16 = 4

Main Effects

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 22	M = 18	Overall M = 20
Self Regulated	M = 18	M = 14	Overall M = 16
	Overall M = 20	Overall M = 16	

Main effect of study time:
20-16 = 4

Interactions

- In the previous example, the effects of one factor were independent of the levels of the second factor
- Neither factor had a direct influence on the other
- The difference between paper versus on-screen presentation did not depend on how study time was controlled
- The main effect for presentation mode (the 4-point difference in test scores) applied equally to both study-time conditions
- There was a 4-point difference between paper and on-screen in the top row (fixed) and in the bottom row (self regulated)

Interactions

- Sometimes, one factor has a direct influence on the effect of a second factor, producing an **interaction between factors**
- An **interaction** occurs whenever two factors, acting together, produce mean differences not explained by the main effects of the two factors
- If the main effect for either factor applies equally across all levels of the second factor, then the two factors are independent, and there is *no interaction*

Interactions

- We illustrate an interaction between factors using a new data set for the paper/on-screen study
- These reflect the actual pattern of results observed in the original study by Ackerman and Goldsmith (2011)

Interactions

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 20	M = 20	Overall M = 20
Self Regulated	M = 20	M = 12	Overall M = 16
	Overall M = 20	Overall M = 16	

Interactions

Study Time	Presentation Format: On Paper	Presentation Format: On Screen	
Fixed	*M* = 20	*M* = 20	Overall *M* = 20
Self Regulated	*M* = 20	*M* = 12	Overall *M* = 16
	Overall *M* = 20	Overall *M* = 16	

Overall *M* = 20 ⟷ Overall *M* = 16

Main effect of presentation format:
20-16 = 4

Interactions

Presentation Format

Study Time	On Paper	On Screen	
Fixed	*M* = 20	*M* = 20	Overall *M* = 20
Self Regulated	*M* = 20	*M* = 12	Overall *M* = 16
	Overall *M* = 20	Overall *M* = 16	

4-point main effect of presentation format does not explain results for fixed study time

Main effect of presentation format: 20-16 = 4

Interactions

Presentation Format

Study Time	On Paper	On Screen	
Fixed	*M* = 20	*M* = 20	Overall *M* = 20
Self Regulated	*M* = 20	*M* = 12	Overall *M* = 16
	Overall *M* = 20	Overall *M* = 16	

Overall *M* = 20 ⟷ Overall *M* = 16

Nor does it explain the results for self-regulated study time

Main effect of presentation format: 20-16 = 4

Interactions

Presentation Format

Study Time	On Paper	On Screen	
Fixed	*M* = 20	*M* = 20	Overall *M* = 20
Self Regulated	*M* = 20	*M* = 12	Overall *M* = 16
	Overall *M* = 20	Overall *M* = 16	

Interactions

Presentation Format

Study Time	On Paper	On Screen	
Fixed	$M = 20$	$M = 20$	Overall $M = 20$
Self Regulated	$M = 20$	$M = 12$	Overall $M = 16$
	Overall $M = 20$	Overall $M = 16$	

Main effect of study time:
20-16 = 4

Interactions

Study Time	Presentation Format: On Paper	Presentation Format: On Screen	
Fixed	M = 20	M = 20	Overall M = 20
Self Regulated	M = 20	M = 12	Overall M = 16
	Overall M = 20	Overall M = 16	

Main effect of study time:
20-16 = 4

4-point main effect of study time does not explain results for on-paper presentation

Interactions

Presentation Format

Study Time	On Paper	On Screen	
Fixed	M = 20	M = 20	Overall M = 20
Self Regulated	M = 20	M = 12	Overall M = 16
	Overall M = 20	Overall M = 16	

Main effect of study time:
20-16 = 4

Nor does it explain the results for on-screen presentation

Interactions

- Earlier, we defined an interaction as *unique mean differences not explained by the main effects*
- An alternative, more common, definition is that an **interaction** exists *when the effects of one factor depend on the different levels of a second factor*
- For the data just examined, in the fixed-time condition, there is no difference between the two presentation formats
- However, in the self-regulated condition, the paper group scores an average of 8 points higher on the test
- Thus, the effect of one factor (presentation format) depends on the levels of the second (study time), which indicates an interaction

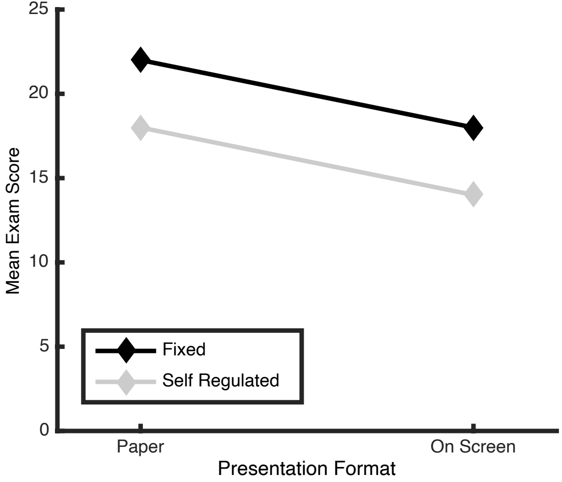
Identifying Interactions in a Data Matrix

- To identify an interaction in a data matrix, we compare the mean differences in any individual row (or column) with the mean differences in other rows (or columns)
- If the size and direction of differences in one row (or column) are the same as the corresponding differences in other rows (or columns) there is no interaction
- If the differences change from one row (or column) to another, there is evidence of an interaction
- For example, in the data just examined, the two means in the top row are 20 and 20, whereas in the bottom row they are 20 and 12
- As the mean difference changes from the top to the bottom row, these data indicate the presence of an interaction

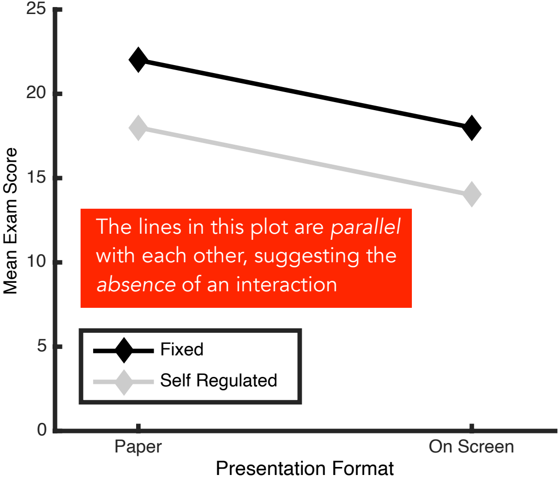
Identifying Interactions in Graphs

- Typically, it is easier to detect the presence or absence of an interaction by plotting the data visually as a line graph
- For a two-factor study, one factor is chosen as the independent variable to appear on the horizontal axis
- Different lines are then plotted, each representing a different level of the second independent variable
- When the results of a two-factor study are graphed, the existence of *nonparallel* lines (lines that cross or converge) is an indication of an interaction between factors

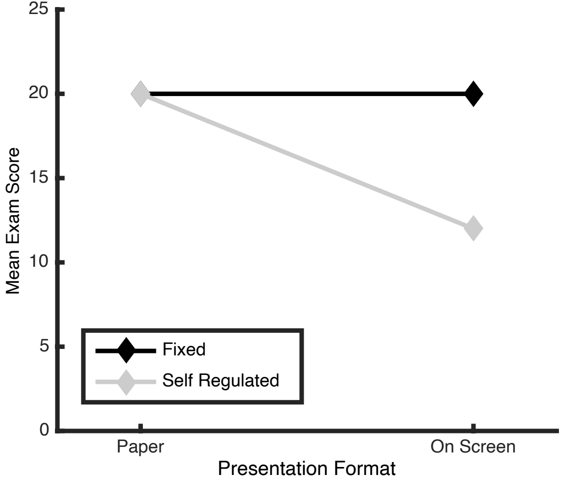
Identifying Interactions in Graphs



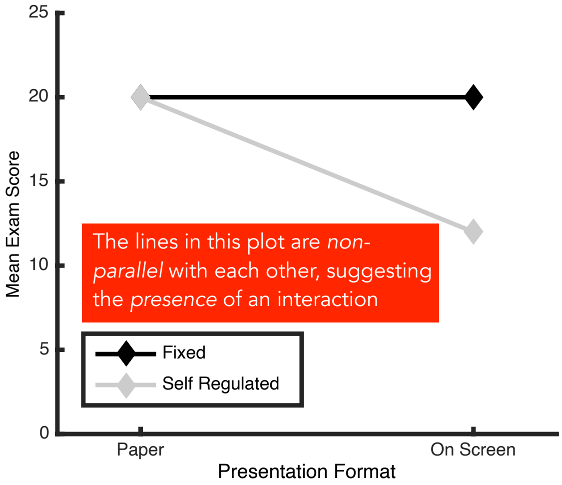
Identifying Interactions in Graphs



Identifying Interactions in Graphs



Identifying Interactions in Graphs



The lines in this plot are *non-parallel* with each other, suggesting the *presence* of an interaction

Interpreting Main Effects and Interactions

- In a two-factor study, mean differences between columns and between rows describe the main effects; mean differences between cells describe the interaction
- However, these mean differences are merely descriptive
- They must be evaluated by a statistical test (discussed later) before they can be considered significant
- Until the data are analysed by statistical test, you should exert caution interpreting the results of a factorial study

Interpreting Main Effects and Interactions

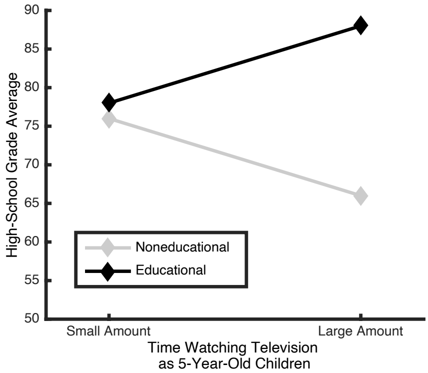
- Even if a statistical analysis reveals significant effects, you must still interpret data cautiously
- In particular, if the analysis yields a significant interaction, then the main effects, whether significant or not, may not present an accurate picture of the data
- Remember, the main effect for one factor is obtained by averaging all the different levels of the second factor
- Since each main effect is an average, it may not accurately represent any of the individual effects used to compute that average

Interpreting Main Effects and Interactions

Relationship between the TV Viewing Habits of 5-Year-Old Children and Their Future High-School Grades.
Based on results from Anderson, Huston, Wright, and Collins (1998).

Viewing Habbits

Type of Programs	Small Amount	Large Amount	
Ed.	*M* = 78	*M* = 88	Overall *M* = 83
Noned.	*M* = 76	*M* = 66	Overall *M* = 71
	Overall *M* = 77	Overall *M* = 77	



Interpreting Main Effects and Interactions

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There is an interaction between TV viewing habits and type of program.

Viewing Habbits / **Type of Programs**

	Small Amount	Large Amount	
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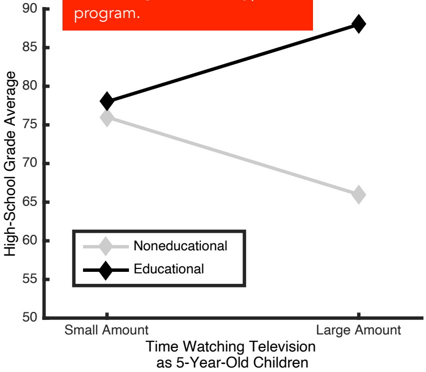
	Overall *M* = 77	Overall *M* = 77	



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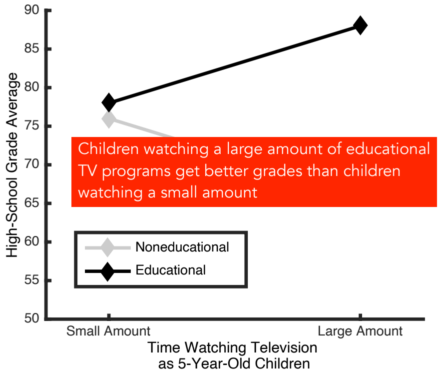

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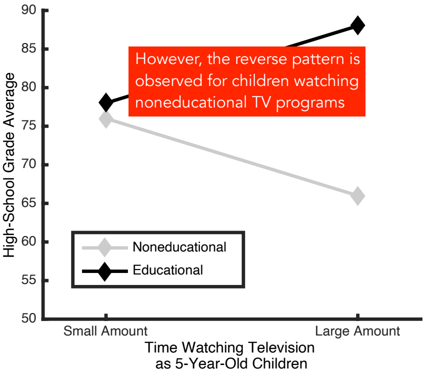

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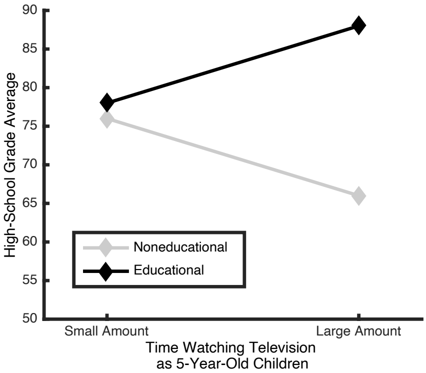

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Viewing Habbits

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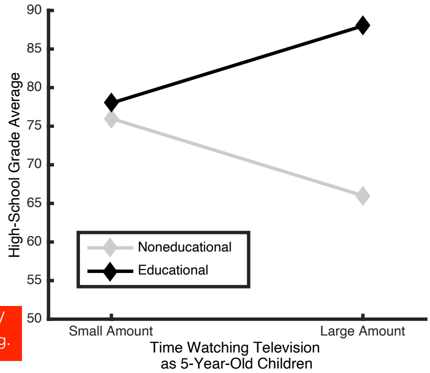
	Overall *M* = 77	Overall *M* = 77	

Concluding from the absence of a main effect of viewing habits that TV viewing has no effect on subsequent grades would be highly misleading.



Independence of Main Effects and Interactions

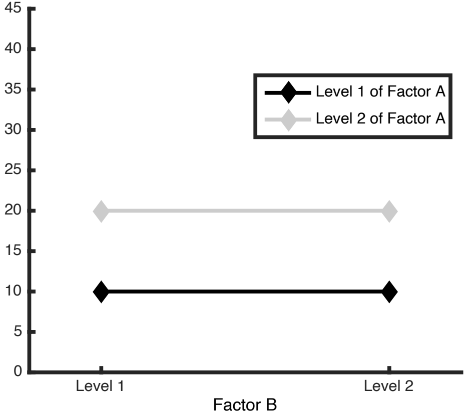
- A two-factor study allows us to evaluate three separate sets of mean differences:
  1. Mean differences from the main effect of factor A
  2. Mean differences from the main effect of factor B
  3. Mean differences from the interaction between factors
- The three sets of mean differences are separate and completely independent
- A two-factor study may therefore yield *any* possible combination of main effects and interaction

Three Possible Combinations of Main Effects and Interactions

Data showing a main effect for factor A but no main effect for factor B and no interaction

Factor B

Factor A			
	M = 20	M = 20	Overall M = 20
	M = 10	M = 10	Overall M = 10
	Overall M = 15	Overall M = 15	

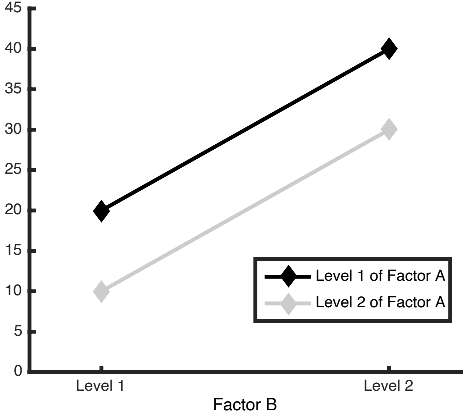


Three Possible Combinations of Main Effects and Interactions

Data showing main effects for both factor A and factor B but no interaction

Factor B

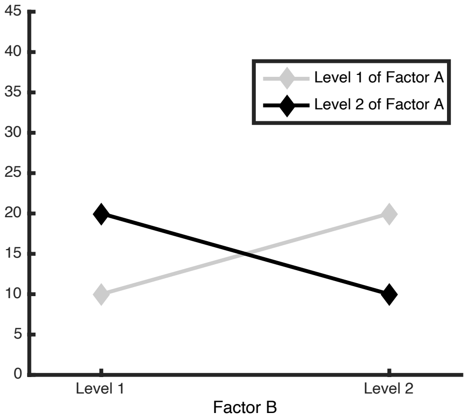
Factor A			
	M = 10	*M* = 30	Overall *M* = 20
	M = 20	*M* = 40	Overall *M* = 30
	Overall *M* = 15	Overall *M* = 35	



Three Possible Combinations of Main Effects and Interactions

Data showing no main effect for either factor, but an interaction

	Factor B		
Factor A	*M* = 10	*M* = 20	Overall *M* = 15
	M = 20	*M* = 10	Overall *M* = 15
	Overall *M* = 15	Overall *M* = 15	



Design Types

Types of Factorial Designs

- There are different types of factorial designs:
  1. Between- and within-participants designs
  2. Mixed designs
  3. Pretest–posttest designs
  4. Higher-order factorial designs

Between- and Within-Participants Designs

- A factorial study can be constructed that is purely a between-participants design
- The advantages and disadvantages of such a design are the same as those highlighted in previous lectures
- One disadvantage merits further comment; specifically, between-participants designs require a large number of participants
- In factorial designs, this problem is often worsened because a multi-factor study typically has more treatment conditions than a single-factor study
- For example, with 30 participants per treatment group a 2×4 factorial design has 8 treatment conditions and requires a total of 240 (8×30) participants

Between- and Within-Participants Designs

- Another disadvantage of between-participants designs is that individual differences can become confounding variables and increase the variance of scores
- On the positive side, a between-participants design is not subject to order effects
- Such designs are best suited to when lots of participants are available, individual differences are small, and order effects are likely

Between- and Within-Participants Designs

- A factorial study can also be constructed that is purely a within-participants design
- The advantages and disadvantages of such a design are the same as those highlighted in previous lectures
- A particular disadvantage for a factorial study is the number of treatment conditions a participant must undergo
- In a 2×4 factorial study, for example, each participant must complete 8 different treatment conditions
- This can be time-consuming, introduce testing effects (e.g., fatigue or practice effects), and make it more difficult to counterbalance the design to control for order effects

Between- and Within-Participants Designs

- On the positive side, within-participants designs require fewer participants and reduce problems associated with individual differences
- Such designs are best suited to situations in which individual differences are large, and there is little reason to expect order effects to be large and disruptive

Mixed Designs

- Sometimes the advantages of a between-participants design apply to one factor, whereas the advantages of a within-participants design apply to another factor
- For example, one might want to use a within-participants design to take maximum advantage of a small group of participants
- However, if one factor is expected to produce large order effects, then a between-participants design should be used for that factor
- A **mixed design** is a factorial design with one between-participants factor and one within-participants factor

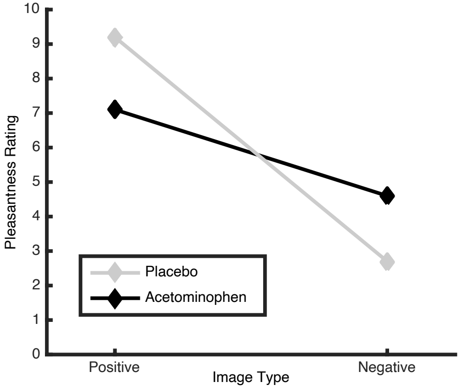
Example: Durso, Luttrell, and Way (2015)

- Examined the effect of acetominophen on experience of pleasure and pain
- Half the participants were given a 1000 mg dose of acetominophen and half were given a placebo (between-participants factor)
- Participants then saw a series of 40 photographs, some containing highly positive images and some containing highly negative images (within-participants factor)
- Participants were required to rate the pleasantness/unpleasantness of each photo

Example: Durso, Luttrell, and Way (2015)

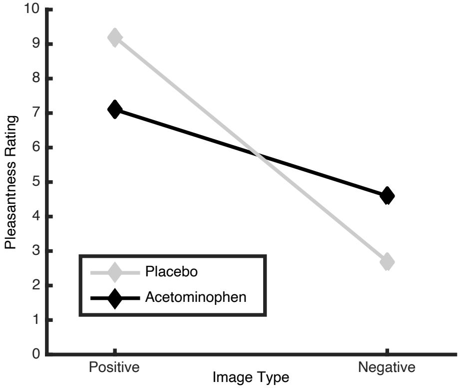
Image Type

Drug Condition	Positive	Negative	
Acetamin.	*M* = 7.1	*M* = 4.6	Overall *M* = 5.85
Placebo	*M* = 9.2	*M* = 2.7	Overall *M* = 5.95
	Overall *M* = 8.15	Overall *M* = 2.65	



Example: Durso, Luttrell, and Way (2015)

Between-Participants Factor	Within-Participants Factor: Positive	Within-Participants Factor: Negative	
Acetamin.	M = 7.1	M = 4.6	Overall M = 5.85
Placebo	M = 9.2	M = 2.7	Overall M = 5.95
	Overall M = 8.15	Overall M = 2.65	



Pre- and Post-Test Control Group Designs

- A pretest–posttest design involves (at least) two groups of participants
- One group, the treatment group, is measured before and after receiving a treatment
- A second group, the control group, is also measured twice (pretest and posttest) but does not receive any treatment between the two measurements
- This design can be represented as follows:

R	O	X	O	(treatment group)
R	O		O	(control group)

- Where O represents a measurement, X represents a treatment, and R represents random assignment

Pre- and Post-Test Control Group Designs

	Pretest	Posttest
Treatment Group	Pretest scores for participantswho receive the treatment	Posttest scores for participants who receive the treatment
Control Group	Pretest scores for participants who do not receive the treatment	Posttest scores for participants who do not receive the treatment

Pre- and Post-Test Control Group Designs

- This design is an example of a two-factor mixed design
- One factor, treatment/control, is a between-participants factor
- The other factor, pretest-posttest, is a within-participants factor

Higher-Order Factorial Designs

- **Higher-order factorial designs** are those that incorporate three or more factors
- Although powerful, such designs introduce additional complexity
- For example, a three-factor design has three factors (A, B, & C) and produces three main effects
- It also generates three two-way interactions $A \times B$, $B \times C$, $A \times C$
- Additionally, the extra factor introduces the potential for a three-way interaction: $A \times B \times C$

Higher-Order Factorial Designs

- A two-way interaction, such as A $\times$ B, indicates that the effect of factor A depends on the levels of factor B
- The A $\times$ B $\times$ C three-way interaction indicates that the two-way interaction between A and B depends on the levels of factor C
- A three-way interaction can be a challenge to interpret, especially if there are more than two levels within a factor
- It is much harder to interpret a four-way (or higher) interaction
- Although it is possible to add factors to a study without limit, studies incorporating more than three factors can yield complex results that are difficult to interpret

Analysis

Statistical Analysis of Factorial Designs

- The analysis of a factorial design is undertaken using factorial ANOVA
- The version used depends on whether the design is between-participants, within-participants, or mixed
- The two-factor ANOVA conducts three separate hypothesis tests:
  - one to evaluate the main effect of factor A
  - one to evaluate the main effect of factor B
  - one to evaluate the interaction
- The test uses an F-ratio to determine whether the actual mean differences in the data are significantly larger than expected by chance

Key Terms and Definitions

- **Factor:** an independent variable in an experiment, especially those that include two or more independent variables.
- **Factorial design:** a research design that includes two or more factors.
- **Main effect:** the mean differences among the levels of one factor. When a study is represented as a matrix with one factor defining the rows and the second factor defining the columns, the mean differences among the rows define the main effect for one factor, and the mean differences among the columns define the main effect for the second factor.
- **Interaction:** occurs whenever two factors, acting together, produce mean differences that are not explained by the main effects of the two factors. An interaction exists when the effects of one factor depend on the different levels of a second factor.
- **Mixed design:** a factorial study that combines two different research designs. A common example of a mixed design is a factorial study with one between-participants factor and one within-participants factor.

Hope You Enjoyed The Segment!