Supplementary material for:

“How is the serial order of a spatial sequence coded? Insights from transposition latencies”

by Hurlstone & Hitch

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Abstract

This supplementary document reports applications of two generalized versions of the five models of serial order to the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). Note that this is not a stand-alone document; please consult the main article before reading this supplement.
Hurlstone and Hitch (submitted) report three experiments involving the output-timed serial recall of sequences of spatial locations that tested the transposition error latency predictions of five alternative models and mechanisms for representing serial order. The results of these experiments consistently support the theoretical predictions of a model in which serial order is represented via a primacy gradient of activations over items, associations between items and position markers, and with suppression of items following recall—the PG+PM+RS model. Qualified support for this model was provided by the results of quantitative fits of the five models to the ungrouped condition of Experiment 3 of Hurlstone and Hitch, which confirmed that only this model could reproduce the latency-displacement function (LDF) observed empirically when model parameters were estimated directly from the behavioral data.

However, due to limitations of space, we only examined the predictions of the models for ungrouped sequences, yet two of our experiments (Experiments 1 & 3) incorporated a temporal grouping condition. Although the form of the LDF observed empirically was generally invariant with respect to this grouping manipulation, this does not preclude the possibility that when the models are applied to grouped sequences they might nevertheless predict different LDFs from those predicted for ungrouped sequences. Accordingly, one objective of the simulations reported here was to apply the five models to the grouped condition of Experiment 3 of Hurlstone and Hitch in order to establish the generality of their transposition error latency predictions. Of critical interest is whether the PG+PM+RS model still provides the best account of the
LDF observed empirically.

A second motivation for the simulations was to try and elucidate the nature of the positional representations supporting grouped spatial sequences. Positional models of verbal serial recall represent grouped sequences on two dimensions, with one dimension representing the position of groups in the sequence overall and with a second dimension representing the position of items within groups (Brown, Preece, & Hulme, 2000; Henson, 1998; Lewandowsky & Farrell, 2008). The latter position-within-group representation of order is crucially necessary for modeling the interposition errors that are observed in recall of grouped verbal sequences (Farrell & Lelièvre, 2009; Henson, 1996, 1999; Ng & Maybery, 2002, 2005; Ryan, 1969). Positional models predict these errors because items in different groups share overlapping within-group positional codes, rendering them vulnerable to temporal confusion.

In Hurlstone and Hitch, we found that temporal grouping exerted a number of systematic effects on spatial serial recall accuracy, latency, and errors that are functionally similar to its effects on verbal serial recall. Specifically, temporal grouping enhanced the accuracy of recall, caused mini primacy and recency effects within-groups, long output times prior to the production of the first item of each sub-group, and a reduction in the frequency of transpositions between groups. These functional similarities notwithstanding, in both experiments, we failed to observe an increase in the frequency of interpositions in grouped spatial sequences (see also Parmentier, Andrés, Elford, & Jones, 2006; Experiment 4 and Hurlstone, 2010; Experiment 10).

How might one account for the various functional similarities between the effects of temporal grouping on verbal and spatial serial recall, whilst at the same time accounting for the one functional difference—interpositions? One possibility is that in the spatial domain people use a different multidimensional representation of positional
information than in the verbal domain, with one dimension representing the positions
of groups and with the second dimension representing the positions of items in the
sequence overall, rather than within-groups. In this supplement, we sought to test the
viability of this hypothesis by fitting versions of the position marking (PM), position
marking and response suppression (PM+RS), position marking and output interference
(PM+OI), and primacy gradient, position marking, and response suppression
(PG+PM+RS) models to the grouped condition of Experiment 3 of Hurlstone and Hitch
using starting activations for position marking based upon the above representational
assumptions for grouped spatial sequences.

Although intuition might suggest that the standard model in which items are
coded for their position within-groups would necessarily predict interposition errors,
the possibility cannot be excluded that there may be portions of such a model’s
parameter space in which it can reproduce the human data showing no elevation in
such errors. Accordingly, we also applied separate versions of the same models to the
data employing starting activations for position marking based upon the standard
position-of-group and position-within-group representation of positional information.

For completeness, we also report applications of two augmented versions of the
primacy gradient and response suppression (PG+RS) model to the same data. Temporal
grouping effects have traditionally been interpreted as falling outside the purview of
such primacy models (Farrell & Lelièvre, 2009; Henson, 1998; Lewandowsky & Farrell,
2002). In order for a primacy model to produce interpositions, it is necessary to adopt a
multidimensional representational scheme within which one primacy gradient
represents the order of groups and a second primacy gradient represents the order of
items within-groups. However, the problem with this representational scheme is that it
predicts that interpositions will always consist of the anticipation of the first item of the
subsequent group, which is at odds with the empirical data on the distribution of
interpositions (Henson, 1996). The problem is illustrated in Figure 3 (right-panel) which shows a primacy gradient for a 9-item sequence grouped into threes that is a weighted combination of a primacy gradient representing the order of groups in the sequence and a primacy gradient representing the order of items within-groups. It can be seen that to produce interpositions a representational scheme must be adopted that forces the model into committing anticipation errors. Consequently, such a model should tend to be unable to ever reproduce a sequence correct in its entirety.

This problem disappears in the context of modeling temporal grouping effects in the spatial domain because, as we have seen, interposition errors are not witnessed in temporally grouped spatial sequences. Thus, the residual effects of grouping might be explicable by a representational scheme in which the primacy gradient representing the order of groups is supplemented by a primacy gradient representing the order of items in the sequence overall (see Figure 3; left-panel). Since such a model should tend not to produce interpositions, it should not suffer from the abovementioned difficulties. However, the possibility should not be excluded that a representational scheme based upon a primacy gradient representing the order of groups complemented by a primacy gradient representing the order of items within-groups might also possess parameter settings under which it can reproduce the key aspects of the grouped data, without predicting interpositions. Accordingly, in the following simulations, we compared the predictions of both versions of the primacy gradient model.

In summary, we sought to test two versions of each of the five models examined in our main article. Both sets of models incorporated one dimension that represented the order of groups in the sequence, but whilst one set of models incorporated a second dimension that represented the order of items in the sequence overall, the other set of models incorporated a second dimension that represented the order of items within-groups. In what follows, we report the results of quantitative fits of the two sets
of models to the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). To anticipate, the results of the simulations revealed that both sets of models accounted for the effects of grouping on the accuracy serial position curve and the absence of interpositions in the associated transposition gradients. In addition, all models predicted LDFs that did not differ qualitatively from those predicted by the versions of the models applied to ungrouped sequences in our original article. As anticipated, both implementations of the PG+PM+RS model—i.e., order-within-sequence and order-within-group—provided the most accurate description of the LDF observed empirically. We subsequently report a parameter sensitivity analysis of the two implementations of the PG+PM+RS model that sought to establish the robustness of the model predictions for interpositions. The results of these analyses revealed that the order-within-sequence implementation of this model predicted interpositions across a minority of its explored parameter settings, indicating that the absence of interpositions is characteristic of this model’s behavior. By contrast, the order-within-groups implementation of this model predicted interpositions across a majority of its explored parameter settings, suggesting that the model’s failure to predict interpositions under its best-fitting parameter settings constitutes an exception to its more general pattern of behavior. The results of the simulations therefore provide qualified support for the hypothesis that items in grouped spatial sequences are coded for their position in the sequence overall rather than their position within-groups.

Quantitative modeling

Implementation of grouped representations

We begin by describing how the starting activations were computed for the different implementations of position marking and a primacy gradient for grouped sequences before describing the modeling procedure and the results of our simulations.
Position marking. We implemented two variants of position marking for grouped sequences. In the position-of-groups and position-within-sequence implementation, starting activations were chosen that directly reflected the confusability of group and item positions in the sequence. Specifically the starting activations were generated using the following equation:

\[ a_j = (1 - \omega) \cdot \lambda \cdot \theta^{|g-l|} + \omega \cdot \lambda \cdot \theta^{|j-r|} \]  

(1)

Where \( j \) indexes an item's input position, \( g \) indexes its group's input position, and \( l \) represents the input position of the group of the target item to-be-recalled at the current response (output) \( r \) position. To explain, suppose we wish to calculate the activation of the fourth item (in a 9-item sequence grouped into threes) at the eighth response position. In this example, \( j \) equals 4, \( g \) equals 2 (since item 4 appears in the second group), \( r \) equals 8, and \( l \) equals 3 (since item 8 appears in the third group). The parameter \( \theta \) governs the distinctiveness of the position marking activations \((0 < \theta \leq 1)\), whilst \( \lambda \) is a scaling parameter \((0 < \lambda \leq 1)\). The first term in equation 1 generates gradients of activations corresponding to the positions of groups, whilst the second term generates gradients of activations corresponding to the positions of items within the sequence. The parameter \( \omega \) is an attentional weighting parameter that permits attention to be differentially allocated to the two positional dimensions \((0 < \omega \leq 1)\). When \( \omega = .5 \), attention is directed equally to the two dimensions; when \( \omega < .5 \), more attention is allocated to the group-position-in-sequence dimension of order; when \( \omega > .5 \), more attention is allocated to the item-position-in-sequence representation of order.

Example starting activations for this representational scheme for all output positions in a 9-item sequence grouped into threes are shown in Figure 1 (note that the starting activations generated at each output position by equation 1 were rescaled to sum to 1, but Figure 1 shows the unscaled activations).
In the position-of-groups and position-within-group implementation of position marking, starting activations were chosen that directly reflected the confusability of group positions in the sequence and item positions within groups. Accordingly, the starting activations were generated as follows:

$$a_j = (1 - \omega) \cdot \lambda \cdot \theta^{g-l} + \omega \cdot \lambda \cdot \theta^{i-p}$$

Where $i$ indexes the within-group input position of item $j$ and $p$ represents the within-group input position of the target item to-be-recalled at the current response position. To elaborate, using the earlier example of calculating the activation of the fourth item at the eighth response position, $i$ would equal 1 (since item 4 appears in the first position in the second group), whilst $p$ would equal 2 (since item 8 appears in the second position in the third group). The first term in equation 2 is the same as in equation 1 and generates gradients of activations corresponding to the positions of groups. The second term generates gradients of activations representing the positions of items within-groups. As in equation 1, the parameter $\omega$ weights the amount of attention allocated to the two positional dimensions. Example starting activations for this representational scheme for all output positions in a 9-item sequence grouped into threes are shown in Figure 2 (as above, the starting activations generated at each output position by equation 2 were rescaled to sum to 1, but Figure 2 shows the unscaled activations).

**Primacy gradient.** As for position marking, we implemented two variants of the primacy gradient for grouped sequences. In the order-of-groups and order-within-sequence implementation, starting activations were computed in the following manner:
\[ a_j = (1 - \omega) \cdot a_1 \cdot \gamma_{g-1} + \omega \cdot a_1 \cdot \gamma_{j-1} \] (3)

Where \( g \) indexes the group of the \( j \)th item, \( a_1 \) is the start value for the primacy gradients \((0 < a_1 \leq 1)\), and \( \gamma \) is a parameter controlling the steepness of the primacy gradients \((0 < \gamma \leq 1)\). The first term in equation 3 generates a primacy gradient representing the order of groups, whereas the second term generates a primacy gradient representing the order of items in the sequence. As above, the parameter \( \omega \) is an attentional weighting parameter that enables attention to be differentially allocated to the two dimensions of order. When \( \omega = .5 \), attention is directed equally to the two dimensions; when \( \omega < .5 \), more attention is allocated to the primacy gradient representing the order of groups; when \( \omega > .5 \), more attention is allocated to the primacy gradient representing the order of items. Example starting activations for this representational scheme for the first output position in a 9-item sequence grouped into threes are shown in Figure 3 (left-panel).

For the order-of-groups and order-within-group implementation of the primacy gradient, the starting activations were determined using equation 4:

\[ a_j = (1 - \omega) \cdot a_1 \cdot \gamma_{g-1} + \omega \cdot a_1 \cdot \gamma_{p-1} \] (4)

Where the first term represents the order of groups, the second term represents the order of items within-groups, and the parameter \( \omega \) weights the attention given to the two dimensions of order. Example starting activations for this representational scheme for the first output position in a 9-item sequence grouped into threes are shown in Figure 3 (right-panel).
Model comparisons

We fitted two different versions of the five models to the accuracy serial position curves and transposition gradients of individual participants for the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). In one set of models, the starting activations for position marking in the PM, PM+RS, PM+OI, and PG+PM+RS models were generated using equation 1, and the starting activations for the primacy gradient in the PG+RS model were generated using equation 3. Thus, in these versions of the models, grouping effects were modeled by assuming that grouped sequences are supported by representations of the order of groups and the order of items within the sequence overall. In the second set of models, the starting activations for position marking in the PM, PM+RS, PM+OI, and PG+PM+RS models were generated using equation 2, and the starting activations for the primacy gradient in the PG+RS model were generated using equation 4. Thus, in these versions of the models, grouping effects were modeled by assuming that grouped sequences are supported by representations of the order of groups and the order of items within-groups. To avoid confusion, note that in the PG+PM+RS models, the starting activations for the primacy gradient were computed as for ungrouped sequences in Hurlstone and Hitch (submitted; equation 3). Thus, only the starting activations for position marking were computed using the generalized equations for grouped sequences introduced in this supplement. The implementation of the remaining representational principles (viz., response suppression, output interference) was the same as for ungrouped sequences in Hurlstone and Hitch (submitted).

Fitting procedure. The fitting procedure was the same as that employed to fit the models to the ungrouped condition of Experiment 3 of Hurlstone and Hitch (submitted). Specifically, the models were fit to the data of individual participants (26 in
total) and their predictions were then averaged to give aggregate predictions. Model parameter values were varied systematically using the simplex algorithm (Nelder & Mead, 1965) in order to minimize the Root Mean Square Deviation (RMSD)—summed across the accuracy serial position curve and transposition gradients—between the data and the prediction of each model, for each individual participant. The RMSD was defined as:

$$RMSD = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (obs_i - pred_i)^2}$$

(5)

Where $obs_i$ is the value $i$ observed in the data, $pred_i$ is the corresponding value predicted by the model, and $N$ is the number of data points. The parameters that were free to vary during the fitting process for the PM and PM+RS models were the distinctiveness of the position markers ($\theta$), the attentional weighting parameter ($\omega$), and the standard deviation of noise applied during the iterative updating process ($\sigma$). The PM+OI models took the same free parameters in addition to the output interference parameter ($\delta$). The parameters that were free to vary for the PG+RS models were the steepness of the primacy gradient ($\gamma$), the attentional weighting parameter ($\omega$), and the standard deviation of noise ($\sigma$). Finally, the free parameters for the PG+PM+RS models were the steepness of the primacy gradient ($\gamma$), the distinctiveness of the position markers ($\theta$), the attentional weighting parameter ($\omega$), and the standard deviation of noise ($\sigma$). The remaining model parameters were fixed and assumed the following values: $\lambda = 1$ (scaling parameter for position marking), $a_1 = 1$ (starting point for primacy gradient), and $\alpha = .95$ (amount of response suppression). The parameters underlying the operation of the lateral inhibition network assumed the same values employed in the main article ($W^+ = 1.1; W^- = -0.1; T = 1$).

In summary, the number of free model parameters was three for the PM, PM+RS,
and PG+RS models, whilst the PM+OI and PG+PM+RS models both incorporated four free parameters. Each parameter vector explored by the search algorithm involved 10,000 model simulation trials of 9-item sequences grouped into threes.

Predictions and discussion. Before inspecting the predictions of the models, a brief description of their goodness-of-fits is in order. The minimized RMSDs of the fits of the order-within-sequence implementations of the models to individual participants can be inspected in Table 1, whilst the corresponding RMSDs for the order-within-group implementations of the models can be inspected in Table 2. For the former set of models, the PM+OI and PG+PM+RS models jointly provided the best fits to the data; the PM+RS and PG+RS models jointly provided the next best fits; whilst the PM model provided the worst fit. For the latter set of models, the PM+OI model provided the best fit to the data; followed closely by the PG+PM+RS model; whilst the PM, PM+RS, and PG+RS models jointly provided the worst fits to the data. In general, the order-within-sequence implementations of the models provided slightly better fits to the data than their order-within-group counterparts but the two versions of the critical model of interest—namely the PG+PM+RS model—provided equally good fits to the data. It is important to acknowledge that the differences in RMSDs between models are only small, rendering it difficult to adjudicate between the models on the basis of their response probability predictions.

Figure 4 shows the aggregate data for the grouped condition of Experiment 3 (with the ungrouped condition included for comparison) of Hurlstone and Hitch (submitted), whilst the aggregate predictions of the order-within-sequence and order-within-group implementations of the models are shown in Figures 5 and 6, respectively. The accuracy serial position curves predicted by the models are shown in Figures 5A and 6A from which it can be seen that both sets of models predicted mini within-group primacy and recency effects, as witnessed empirically (Figure 4A). The transposition gradients
predicted by the models are illustrated in Figures 5B and 6B from which it can be seen that the models generally predict a sharp drop in response probabilities between absolute transposition displacement values 2 and 3 (this drop being particularly pronounced for the PG+RS models). Further analysis of the predictions of the models revealed that this was due to their tendency to over-predict the frequency of transpositions within-groups. Of critical interest, however, is whether the models—notably the order-within-group instantiations of the models—predicted interpositions. This would be reflected by discontinuities in the transposition gradients, with local peaks at displacements $\pm 3$ and $\pm 6$. However, consistent with the data (Figure 4B), it is visible from inspection of Figures 5B and 6B that the models did not exhibit a tendency to produce interpositions. Figures 5C and 6C display the latency serial position curves predicted by the two sets of models from which it can be seen that the models miss out on the long recall latency preceding the production of the first item in the sequence and each subsequent group (Figure 4C). As noted in our main article, these effects are beyond the purview of the current modeling framework and require additional assumptions about preparatory processes that precede the production of the first item or group in a sequence.

The LDFs predicted by the two sets of models can be inspected in Figures 5D and 6D. Mimicking their transposition gradients (Figures 5B and 6B), the models generally predict a relatively large increase in the mean recall latency at displacement -3, compared to -2, whilst the positional models (PM, PM+RS, and PM+OI) additionally predict a relatively large increase in the mean recall latency at displacement +3 compared to +2. The LDFs predicted by both sets of models are otherwise qualitatively similar to those predicted by the models for ungrouped sequences in Hurlstone and Hitch (submitted). As before, all models predict that the slope of the LDF for anticipations is negative, but the models make different predictions concerning the
slope of the LDF for postponements. As for ungrouped sequences, the PM, PM+RS, and PM+OI models all predict steep positive postponement slopes; the PG+RS models predict a negative postponement slope; whilst the PG+PM+RS models predict a flat postponement slope. Thus, the results of the simulations confirm that when the models are augmented to account for grouping effects, the relative slopes of their predicted LDFs for anticipations and postponements remain qualitatively unaffected.

Robustness of predictions

The results of the quantitative model fitting exercise confirmed that when the five models are augmented to account for the recall of grouped sequences, the PG+PM+RS model still provides the best account of the empirically observed LDF. However, both the position-within-sequence and the position-within-group implementations of this model provided an equally good fit to the data. Notably, both versions of the model predicted the absence of interpositions in their aggregate transposition gradients. Given the comparable descriptive accuracy of the models, how might we adjudicate between them? It has now been established that the absence of interpositions in grouped spatial sequences is a robust feature of the spatial serial recall data, this outcome having now been observed in two of our own experiments (Hurlstone & Hitch, submitted; Experiments 1 & 2), in an experiment reported by Parmentier et al. (2006; Experiment 4) and an unpublished experiment reported by Hurlstone (2010; Experiment 10).

Accordingly, given the consistency of the data, the preferred model should be the one that—across broad variations in its parameter settings—predicts interpositions the least frequently.

To determine the robustness of the predictions of the models for interpositions, we subjected the two versions of the PG+PM+RS model to a parameter sensitivity analysis. As per the sensitivity analyses reported in our main article, we varied the
parameters of each model from .05 to .95 in steps of 0.1 and factorially combined these values to create a grid of parameter setting combinations to-be-explored by simulation. For each model, we varied five parameters, namely the distinctiveness of the position markers $\theta$, the scaling of the position marking activations $\lambda$, the attentional weight given to the two positional dimensions $\omega$, the starting point for the primacy gradient $a_1$, and the slope of the primacy gradient $\gamma$ (the standard deviation of noise $\sigma$ was set to a constant value of .04). To keep the number of simulations manageable, we did not vary the parameter settings for the response suppression parameter $\alpha$, which was instead set to a constant value of .95.\(^2\) The dependent measure of interest was the proportion of simulations on which the models predicted interpositions. For each simulation, the probability of transpositions as a function of (absolute) transposition distance was calculated. A model was classified as predicting interpositions on a given simulation, if the probabilities of transpositions at distances 3 or 6 were larger than the probability of transpositions at distances 2 or 5, respectively.\(^3\) For each parameter vector explored, model predictions were generated for 1000 simulation trials of 9-item sequences grouped into threes.

As anticipated, the results of the sensitivity analysis confirmed that the position-within-group implementation of the PG+PM+RS model is more likely to generate interpositions than its position-within-sequence counterpart: the former model predicted interpositions on 58% of its explored parameter settings, whereas the latter model predicted interpositions on only 3.5% of its explored parameter settings.

### General Discussion

The results of the quantitative model fitting exercise are straightforward and can be summarized as follows. The two versions of the five models examined each accounted for the effect (or lack thereof) of grouping on response probabilities: all
models predicted the mini within-group primacy and recency effects witnessed in the accuracy serial position curve, as well as the absence of interpositions in the associated transposition error gradients. The two versions of the five models also predicted LDFs that are qualitatively similar to those predicted by the restricted versions of the models applied to ungrouped sequences in our main article. Specifically, the PM, PM+RS, and PM+OI models all predicted aggregate LDFs with positive postponement slopes; the PG+RS models predicted aggregate LDFs with negative postponement slopes; while the PG+PM+RS models predicted aggregate LDFs with flat postponement slopes. The results of the simulations therefore confirm that the error latency predictions of the models applied to ungrouped sequences in our main article generalize to conditions of temporal grouping. Furthermore, they confirm that the PG+PM+RS model provides the most accurate description of the LDF observed for both ungrouped and grouped spatial sequences. However, the version of the PG+PM+RS model representing the position of items within-groups and the version of the model representing the position of items in the sequence overall both predicted comparable LDFs, and both models provided equally good fits to the accuracy serial position curves and transposition gradients, rendering it impossible to adjudicate between them. A parameter sensitivity analysis confirmed that the version of the model representing the positions of items within-groups predicts interpositions much more frequently across its parameter space than the version representing the positions of items within the sequence overall. Accordingly, given that the absence of interpositions in grouped spatial sequences appears to be a robust feature of the extant empirical data, these results confer support for the latter—position-within-sequence—version of the model, although only tentatively so.

That the results of the sensitivity analysis only tentatively support this conclusion stems from the fact that we cannot exclude the possibility that interpositions might
manifest in grouped spatial sequences under conditions that are different from those examined in our experiments. One factor that may be particularly crucial for the manifestation of interpositions is the precise construction of the spatial sequences that people must recall. As noted in our main article, although temporal factors exert a strong effect on error production in spatial serial recall tasks—as indicated by the detrimental effect of sequence length on recall accuracy (Smyth & Scholey, 1994; Smyth, 1996), the locality constraint underlying transpositions (Parmentier et al., 2006; Smyth & Scholey, 1996), and the effects of temporal grouping (Hurlstone, 2010; Hurlstone & Hitch, submitted; Parmentier et al., 2006)—it is well-known that spatial constraints exert an effect also. Specifically, properties of spatial sequences such as the relative distance between successive locations (Guérard, Tremblay, & Saint-Aubin, 2009; Parmentier et al., 2006), the number of crosses in the sequence path (Parmentier & Andrés, 2006; Parmentier et al., 2005), the configuration of locations (Kemps, 1999), and the extent to which they can be segregated into sub-groups based on Gestalt organizational principles (De Lillo, 2004; De Lillo & Lesk, 2010; Kemps, 2001; Parmentier et al., 2005; Rossi-Arnaud, Pieroni, & Baddeley, 2006) all exert effects on error production. It seems reasonable to speculate that such spatial constraints—which were not controlled in our experiments—may have interacted in unanticipated ways with our temporal grouping manipulation, causing a shift in the expected pattern of recall errors. Accordingly, further experiments that systematically manipulate different properties of the spatial sequences that must be recalled will be required in order to establish whether the absence of interpositions in grouped spatial sequences is a robust feature of the recall of temporally grouped spatial sequences.

One further aspect of the simulations that briefly merits comment concerns the fits of the PG+RS models. As noted earlier, a common criticism leveled at such primacy models is that they cannot account for grouping effects (Farrell & Lelièvre, 2009;
Henson, 1998; Lewandowsky & Farrell, 2002). However, this criticism only applies if one of the effects of grouping that a primacy model must explain are interpositions. Primacy models encounter difficulties explaining these errors because they predict that interpositions will tend to always involve the anticipation of the first item of the subsequent group—a pattern at odds with the empirical data (Henson, 1996). The inability of primacy models to accurately model interpositions then has knock-on effects on their ability to reproduce other effects of grouping. In the verbal domain, the prevalence of interpositions in grouped verbal sequences therefore rules these models out as accounts of grouping phenomena in this domain. However, the absence of these errors in grouped spatial sequences mean that this limitation poses no such barrier in the spatial domain. Indeed, both versions of the PG+RS model employed in the current model comparisons were able to accurately reproduce the mini primacy and recency effects that are a hallmark characteristic of the accuracy serial position curve for grouped sequences. Nevertheless, the simulations also exposed a key weakness in both models in the form of their predicted transposition gradients: the models predicted a sharp decrease in the probability of transpositions with an absolute displacement value of 3 compared to the probability of transpositions with an absolute displacement value of 2—a pattern attributable to the tendency of both models to dramatically over-predict the frequency of transpositions within-groups. It is also apparent from comparisons of the LDFs predicted by the primacy models with the empirical data that a simple primacy model of grouping is insufficient to accurately reproduce this aspect of the data without also incorporating position markers.

In summary, the results of the current modeling exercise support the hypothesis that positional information in grouped spatial sequences is represented on two dimensions, with one dimension representing the positions of groups and with the second dimension representing the positions of items in the sequence overall. This
representational scheme differs from that required for the accurate modeling of grouping effects in the verbal domain, where information about the positions of groups in the sequence is combined with information about the positions of items within those groups. As such, these results point to a subtle yet fundamental difference between the representation of positional information in verbal and spatial short-term memory. This conclusion is only tentative, however, because it remains to be seen whether interpositions might manifest in grouped spatial sequences when features of the sequences to-be-recalled are controlled more rigorously.
References


Author Note

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Footnotes

1 To avoid excessive model comparisons, we do not report implementations of the PG+PM+RS model employing a grouped version of the primacy gradient. However, simulations of this more complex model revealed that it neither provided a better fit to the response probability data nor a better description of the observed LDF than the version of the model using grouped representations for position marking only. Similarly, we also examined the predictions of a version of the PG+PM+RS model using a grouped primacy gradient and ungrouped position markers and found that this model provided a poorer description of the observed LDF than the model examined here.

2 To put this into context, the number of simulations based on a factorial combination of five parameters with 10 settings each is 100,000 ($10^5$). With six parameters, the number of simulations increases to 1,000,000 ($10^6$).

3 We restricted our analysis to those simulations in which the accuracy of recall at each serial position was 25% or greater. This was to ensure that interpositions were examined under parameter settings associated with realistic levels of recall performance.
Table 1

Minimized RMSDs for the fits of the order-of-groups and order-within-sequence implementations of the five models to the accuracy serial position curves and transposition gradients for the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). The RMSDs of the best fitting model for each individual participant, as well as the best fitting model overall, are indicated in bold.

<table>
<thead>
<tr>
<th>Participant</th>
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<th>PM+OI</th>
<th>PG+RS</th>
<th>PG+PM+RS</th>
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Mean RMSD  0.09  0.08  **0.07**  0.08  **0.07**
Table 2

Minimized RMSDs for the fits of the order-of-groups and order-within-group implementations of the five models to the accuracy serial position curves and transposition gradients for the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). The RMSDs of the best fitting model for each individual participant, as well as the best fitting model overall, are indicated in bold.

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Mean RMSD  0.10  0.10  **0.06**  0.10  0.07
Figure Captions

**Figure 1.** Example starting activations for the position-of-groups and position-within-sequence implementation of position marking for a 9-item sequence grouped into threes. *Note*—the activations were generated using the following parameter values: $\lambda = 1; \theta = .65; \omega = .5$.

**Figure 2.** Example starting activations for the position-of-groups and position-within-group implementation of position marking for a 9-item sequence grouped into threes. *Note*—the activations were generated using the following parameter values: $\lambda = 1; \theta = .65; \omega = .5$.

**Figure 3.** Example starting activations for the order-of-groups and order-within-sequence implementation of the primacy gradient (left-panel) and the order-of-groups and order-within-group implementation of the primacy gradient (right-panel) for the first output position of a 9-item sequence grouped into threes. *Note*—the activations were generated using the following parameter values: $a_1 = .6; \gamma = .85; \omega = .5$.

**Figure 4.** Response probability and recall latency data for the ungrouped and grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). Panels show accuracy serial position curves (A), transposition gradients (B), latency serial position curves (C), and latency-displacement functions (D).

**Figure 5.** Fits of the order-within-sequence implementations of the five models to the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). Panels show accuracy serial position curves (A), transposition gradients (B), latency serial position curves (C), and latency-displacement functions (D).
Figure 6. Fits of the order-within-group implementations of the five models to the grouped condition of Experiment 3 of Hurlstone and Hitch (submitted). Panels show accuracy serial position curves (A), transposition gradients (B), latency serial position curves (C), and latency-displacement functions (D).
Transposition latency supplement, Figure 1
Transposition latency supplement, Figure 2
Transposition latency supplement, Figure 3

Order-Within-Groups

Order-Within-Sequence
**Transposition latency supplement, Figure 4**

**A** *Accuracy SPC*

![Accuracy SPC graph](image)

**B** *Transposition Gradients*

![Transposition Gradients graph](image)

**C** *Latency SPC*

![Latency SPC graph](image)

**D** *Transposition Latencies*

![Transposition Latencies graph](image)
Transposition latency supplement, Figure 5

A  Accuracy SPC

B  Transposition Gradients

C  Latency SPC

D  Transposition Latencies

Legend:
- PM
- PM+RS
- PM+OI
- PG+RS
- PG+PM+RS

Vertical axis:
- Accuracy SPC: Proportion Correct
- Latency SPC: Mean Latency (iterations)

Horizontal axis:
- Serial Position
- Transposition Displacement

Graphs show performance metrics across different conditions and serial positions, illustrating transposition latency and accuracy gradients.
Transposition latency supplement, Figure 6

A  Accuracy SPC

\[ \text{Proportion Correct} \]

B  Transposition Gradients

\[ \text{log(Proportion Responses)} \]

C  Latency SPC

\[ \text{Mean Latency (iterations)} \]

D  Transposition Latencies

\[ \text{Mean Latency (iterations)} \]